Watersheds and Mosaic Images

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Introduction

Importance of the altitude of the pass between basins of the watershed

- Waterfall [Beucher]
- Watershed contours saliency [Najman and Schmitt]

Plan

Minima extension

A *minima extension of an image F* is a subset X of E such that:

- each connected component of X contains one and only one regional minimum of F, and
- each regional minimum of F is included in a connected component of X.

The complementary of a minima extension of F is called a *divide set of F*.

Mosaic image

Let X be a minima extension of an image F. The *mosaic image* of F associated with X is the map $F_X \in \mathcal{F}(E)$ such that

- for any $x \notin X$, $F_X(x) = F(x)$; and
- for any $x \in X$, $F_X(x) = \min\{F(y) | y \in C_x\}$, where C_x denotes the connected component of X that contains x.

Example of minima extension and mosaic image

0	1	2	3	2	1	1
1	2	3	4	3	2	1
2	3	4	5	4	3	2
3	4	5	6	5	4	3
2	3	4	5	4	3	2
2	2	3	4	3	2	1
2	2	2	3	2	1	0

Image

1	1	1	0	1	1	1
1	1	1	0	1	1	1
1	1	1	0	1	1	1
0	0	0	0	0	0	0
1	1	1	0	1	1	1
1	1	1	0	1	1	1
1	1	1	0	1	1	1

Minima extension

0	0	0	3	1	1	1
0	0	0	4	1	1	1
0	0	0	5	1	1	1
3	4	5	6	5	4	3
2	2	2	5	0	0	0
2	2	2	4	0	0	0
2	2	2	3	0	0	0

Mosaic

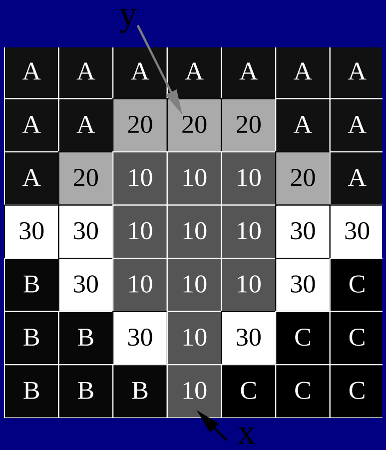
Meyer's watershed algorithm: the extension-by-flooding

1. Attribute to each regional minimum a distinct label; mark each point belonging to a regional minimum with the label of the regional minimum it belongs to. Let Q be a set (initially empty).
2. Insert every non-marked neighbor x of every marked point in the set Q;
3. Extract a point x from the set Q which has the minimal altitude, that is, a point x such that $F(x) = \min\{F(y) | y \in Q\}$. If all marked points in $\Gamma(x)$ have the same label, then
   - Mark x with this label; and
   - For each neighbor y of x such that y is not yet marked and y is not in the set Q, insert y in Q;
4. Repeat step 3 until the set Q is empty.

Extension by flooding: an example

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	10	10	10	20	2
30	30	10	10	10	30	30
1	30	10	10	10	30	0
1	1	30	10	30	0	0
1	1	1	10	0	0	0

Extension by flooding: step 1



Extension by flooding: step 2

A	A	A	A	A	A	A
A	A	20	A	20	A	A
A	20	10	10	10	20	A
30	30	10	10	10	30	30
B	30	10	10	10	30	C
B	B	30	10	30	C	C
B	B	B	10	C	C	C

Extension by flooding: step 3

A	A	A	A	A	A	A
A	A	20	A	20	A	A
A	20	A	A	A	20	A
30	30	A	A	A	30	30
B	30	A	A	A	30	C
B	B	30	A	30	C	C
B	B	B	10	C	C	C

Extension by flooding: step 4

A	A	A	A	A	A	A
A	A	A	A	A	A	A
A	A	A	A	A	A	A
30	30	A	A	A	30	30
B	30	A	A	A	30	C
B	B	30	A	30	C	C
B	B	B	10	C	C	C

Extension by flooding: step 5

A	A	A	A	A	A	A
A	A	A	A	A	A	A
A	A	A	A	A	A	A
30	A	A	A	A	A	30
B	30	A	A	A	30	C
B	B	30	A	30	C	C
B	B	B	10	C	C	C

Extension by flooding: minima extension

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	10	10	10	20	2
30	30	10	10	10	30	30
1	30	10	10	10	30	0
1	1	30	10	30	0	0
1	1	1	10	0	0	0

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
0	1	1	1	1	1	0
1	0	1	1	1	0	1
1	1	0	1	0	1	1
1	1	1	0	1	1	1

Extension by flooding: mosaic image

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	10	10	10	20	2
30	30	10	10	10	30	30
1	30	10	10	10	30	0
1	1	30	10	30	0	0
1	1	1	10	0	0	0

2	2	2	2	2	2	2
2	2	2	2	2	2	2
2	2	2	2	2	2	2
30	2	2	2	2	2	30
1	30	2	2	2	30	0
1	1	30	2	30	0	0
1	1	1	10	0	0	0

(Greyscale) minima extension

Let F and G in $\mathcal{F}(E)$ such that $G \leq F$.
We say that G is a *minima extension (of F)* if:
i) the set composed by the union of all the minima of G is a minima extension of F.
ii) for any $X \in \mathcal{M}(F)$ and $Y \in \mathcal{M}(G)$ such that $X \subseteq Y$, we have $F(X) = G(Y)$.

Extension by flooding: mosaic image

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	10	10	10	20	2
30	30	10	10	10	30	30
1	30	10	10	10	30	0
1	1	30	10	30	0	0
1	1	1	10	0	0	0

2	2	2	2	2	2	2
2	2	2	2	2	2	2
2	2	2	2	2	2	2
30	2	2	2	2	2	30
1	30	2	2	2	30	0
1	1	30	2	30	0	0
1	1	1	10	0	0	0

The mosaic is a minima extension

Extension by flooding: minima extension

15	15	15	15	15	15	15
15	15	20	20	20	15	15
15	20	10	10	10	20	15
30	30	10	10	10	30	30
1	30	10	10	10	30	0
1	1	30	10	30	0	0
1	1	1	10	0	0	0

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
0	1	1	1	1	1	0
1	0	1	1	1	0	1
1	1	0	1	0	1	1
1	1	1	0	1	1	1

Extension by flooding: mosaic image

15	15	15	15	15	15	15
15	15	20	20	20	15	15
15	20	10	10	10	20	15
30	30	10	10	10	30	30
1	30	10	10	10	30	0
1	1	30	10	30	0	0
1	1	1	10	0	0	0

10	10	10	10	10	10	10
10	10	10	10	10	10	10
10	10	10	10	10	10	10
30	10	10	10	10	10	30
1	30	10	10	10	30	0
1	1	30	10	30	0	0
1	1	1	10	0	0	0

The mosaic is NOT a minima extension

Pass value

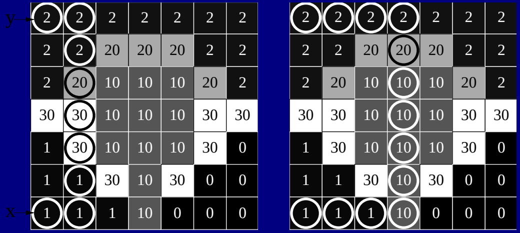
Let F an image. Let $\pi = (x_0, \ldots, x_n)$ be a path in the graph (E, Γ), we set $F(\pi) = \max\{F(x_i) | i = 0, \ldots, n\}$
Let x, y be two points of E, the *pass value for F between x and y* is defined as

$$F(x, y) = \min\{F(\pi) | \pi \in \Pi(x, y)\}$$

where $\Pi(x, y)$ is the set of all paths from x to y.

Let X, Y be two subsets of E which are flat for F, the *pass value for F between X and Y* is defined by $F(X, Y) = F(x, y)$, for any $x \in X$ and any $y \in Y$.

Pass value: illustration



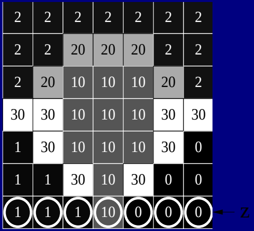
(a) (b)

(a): π_1 from x to y, $F(\pi_1) = 30$.

(b): π_2 from x to$y$, $F(\pi_2) = 20$.

$F(x, y) = 20$

Pass value: illustration



π_3 from x to z, $F(\pi_3) = 10$.

$F(x, z) = 10$

Separation

Let F and image, let $x, y \in E$, let $G \leq F$.

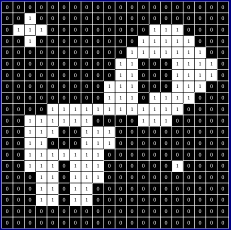
- We say that x and y are *separated (for F)* if $F(x, y) > \max\{F(x), F(y)\}$.
- We say that x and y are *k−separated (for F)* if they are separated for F and if $k = F(x, y)$.
- We say that G is a *separation of F* if, for all x and y in E, whenever x and y are k-separated for F, x and y are k-separated for G.

Summary

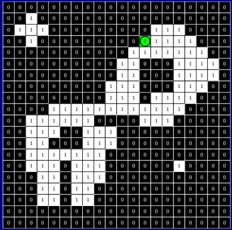
- A mosaic produced by the flooding algorithm is not always a minima extension of the original map.
- A mosaic produced by the flooding algorithm, even in the case where it is a minima extension, is not necessarily a separation of the original map.

W-Simple point (Couprie-Bertrand 1997)

Let $X \subseteq E$.
The point $x \in X$ is *W-simple (for X)* if x is adjacent to one and only one connected component of $\overline{X}$.



Image

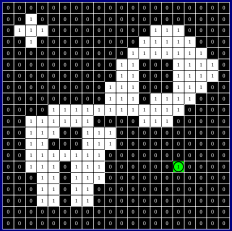


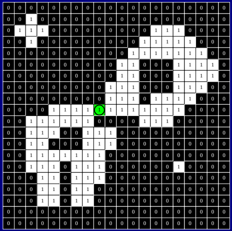
A simple point

A non-simple point

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	1	1	0
0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0	1	1	1	1	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0
0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	1	1	1	0	0	0	0	0	0	1	0	0	0	0
0	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Two examples of simple points





Topological Watershed (Couprie-Bertrand 1997)

Let $F \in \mathcal{F}(E)$, $x \in E$, and $k = F(x)$.
We set $F_k = \{x | F(x) \geq k\}$

- The point x is *W-destructible (for F)* if x is W-simple for F_k.
- We say that $G \in \mathcal{F}(E)$ is a *W-thinning of F* if $G = F$ or if G may be derived from F by iteratively lowering W-destructible points by one.
- We say that $G \in \mathcal{F}(E)$ is a *topological watershed of F* if G is a W-thinning of F and if there is no W-destructible point for G.

Topological watershed

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	10	10	10	20	2
30	30	10	10	10	30	30
1	30	10	10	10	30	0
1	1	30	10	30	0	0
1	1	1	10	0	0	0

Topological watershed

A	A	A	A	A	A	A
A	A	A	A	A	A	A
A	A	A	A	A	A	A
30	30	A	A	A	30	30
A	30	A	A	A	30	A
A	A	30	A	30	A	A
A	A	A	A	A	A	A

Topological watershed

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	10	10	10	20	2
30	30	10	10	10	30	30
1	30	10	10	10	30	0
1	1	30	10	10	0	0
1	1	1	10	0	0	0

Topological watershed

A	A	A	A	A	A	A
A	A	20	20	20	A	A
A	20	10	10	10	20	A
30	30	10	10	10	30	30
B	30	10	10	10	30	C
B	B	30	10	10	C	C
B	B	B	10	C	C	C

Topological watershed

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	10	10	10	20	2
30	30	10	10	10	30	30
1	30	10	10	10	30	0
1	1	30	10	0	0	0
1	1	1	10	0	0	0

Topological watershed

A	A	A	A	A	A	A
A	A	A	A	A	A	A
A	A	A	A	A	A	A
30	30	A	A	A	30	30
A	30	A	A	A	30	A
A	A	30	A	A	A	A
A	A	A	A	A	A	A

Topological watershed

A	A	A	A	A	A	A
A	A	20	20	20	A	A
A	20	B	B	B	20	A
20	30	B	B	B	30	30
B	30	B	B	B	30	B
B	B	30	B	B	B	B
B	B	B	B	B	B	B

Topological watershed

A	A	A	A	A	A	A
A	A	20	20	20	A	A
A	20	B	B	B	20	A
20	30	B	B	B	30	30
B	30	B	B	B	30	B
B	B	30	B	B	B	B
B	B	B	B	B	B	B

Topological watershed

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	1	10	0	20	2
20	1	1	10	0	0	20
1	1	1	10	0	0	0
1	1	1	10	0	0	0
1	1	1	10	0	0	0

Topological watershed and mosaic image

0	1	2	3	2	1	1
1	2	3	4	3	2	1
2	3	4	5	4	3	2
3	4	5	6	5	4	3
2	3	4	5	4	3	2
2	2	3	4	3	2	1
2	2	2	3	2	1	0

Image

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	1	10	0	20	2
20	1	1	10	0	0	20
1	1	1	10	0	0	0
1	1	1	10	0	0	0
1	1	1	10	0	0	0

Topological watershed

2	2	2	2	2	2	2
2	2	20	20	20	2	2
2	20	1	10	0	20	2
30	1	1	10	0	0	30
1	1	1	10	0	0	0
1	1	1	10	0	0	0
1	1	1	10	0	0	0

Mosaic

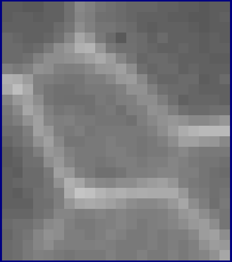
Equivalence Mosaic - Separation

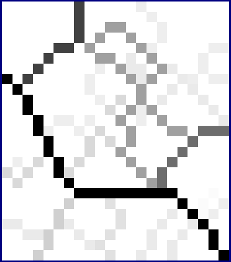
Theorem

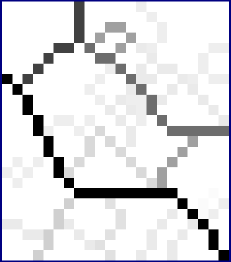
Let $F \in \mathcal{F}(E)$, let X be a minima extension of F, and let F_X be the mosaic of F associated with X. Then F_X is a separation of F if and only if F_X is a W-thinning of F.

Conclusion

- Saliency and Waterfall algorithms


Original image


Saliency with wrong pass values


Saliency with correct pass values

- Thin watersheds