Grayscale watersheds on perfect fusion graph

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Problems

- Region merging methods consist of improving an initial segmentation by iteratively merging pairs of neighboring regions.

T.Pavlidis. *Structural Pattern Recognition*, chapters 4-5. Segmentation techniques, 1977.

Introduction

In mathematical morphology *hierarchical methods* (saliency [NAJMAN96], waterfall [BEUCHER94]) are based on:

- *watershed segmentation*; and
- iterative *merging of the obtained regions*.

Problem 1: grayscale watershed

Problem

Altitudes of passes *between the regions of the watersheds are fundamental for region merging methods based on morphology.*

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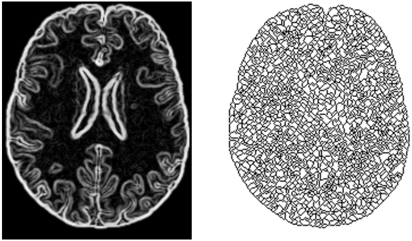
Altitudes of passes *between the regions of the watersheds are fundamental for region merging methods based on morphology.*

- Only topological based watersheds (*W-thinnings*) [NAJMAN05, BERTRAND05] produce divides correctly placed with respect to the altitude of the pass.

Problem 2: region merging



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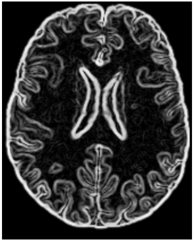

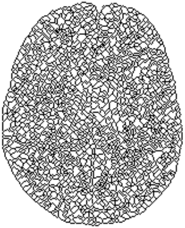

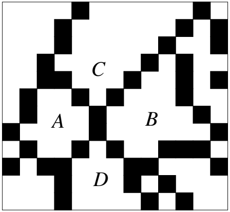

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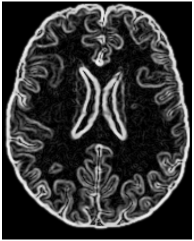

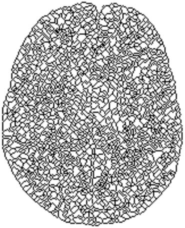

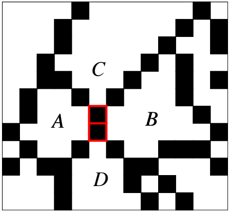

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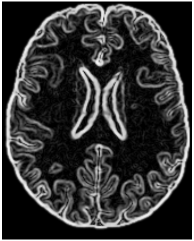

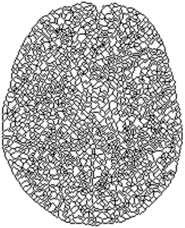

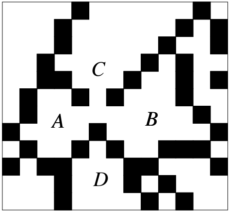

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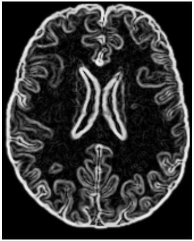

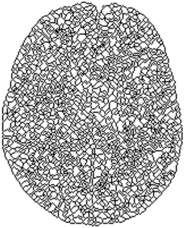

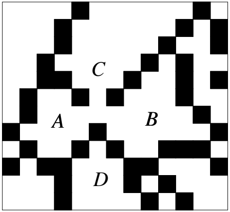

Problem

Is there some graphs in which any pair of neighboring regions can always be merged?

Problem 3: grayscale watersheds & region merging

- Some grayscale watershed algorithms can sometimes produce *thick divides*.
- This is a problem for region merging.

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- This is a problem for region merging.

Problem

- *Is there a class of graphs in which any grayscale watershed is thin?*
- *How is it linked with region merging?*

Grayscale watersheds on perfect fusion graphs

Basic notion on graphs

Let (E, Γ) be a symmetric graph. Let $X \subseteq E$, and let $Y \subseteq X$.

- We say that *X is connected* if $\forall p \in X, q \in X$, there exists a path in X, from p to q.
- We say that *Y is a connected component of X* if Y is both connected and maximal for this property.

- A set X separates its complementary set ($\overline{X}$) into connected components that we call *regions for X*.

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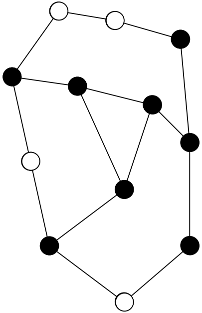

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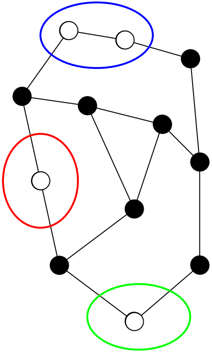

Watershed set: a model of frontier

Let $X \subseteq E$ and $p \in X$

- We say that p is *W-simple for X* if p is adjacent to exactly one region for X.

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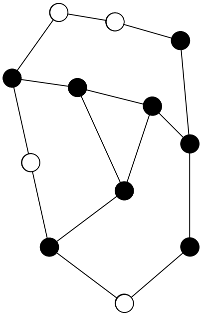

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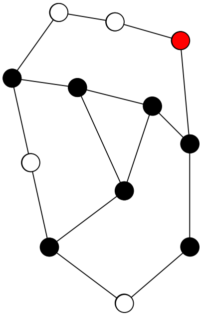

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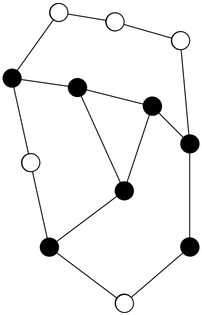
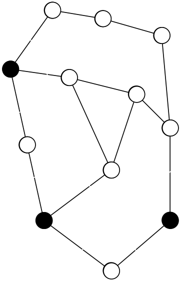


Watershed set: a model of frontier

- The set X is a *watershed set* if there is no W-simple point for X.

Watershed set: example



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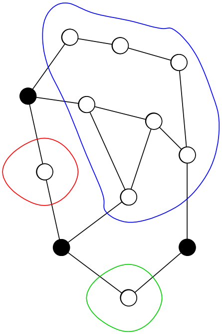
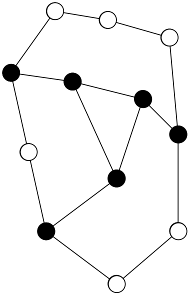
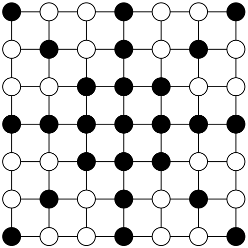
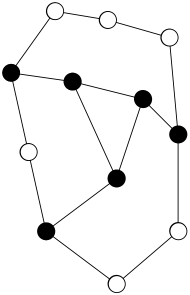


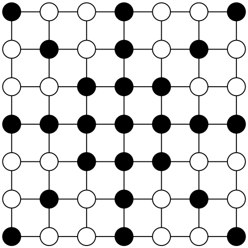
Watershed set: examples





Watershed set: examples





Problem

A watershed set can be thick.

Thin sets

Let $X \subseteq E$.

- We say that X is *thin* if any point in X is adjacent to at least one region for X

Region merging

Let $X \subseteq E$ and let A and B be two regions for X with $A \neq B$.

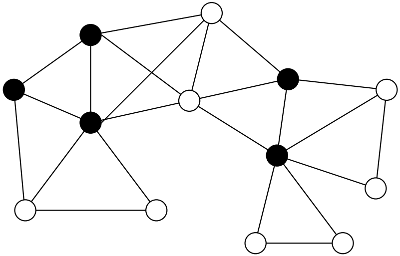
Definition

We say that A and B can be merged (for X) *if there exists* $S \subseteq X$ *such that :*

- *A and B are the only regions for X adjacent to S; and*
- *S is connected.*

We also say that A and B can be merged through S.

Region merging: example



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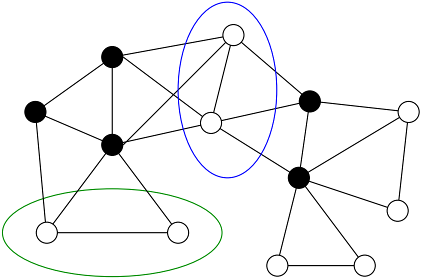

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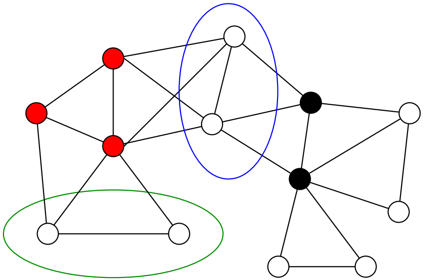

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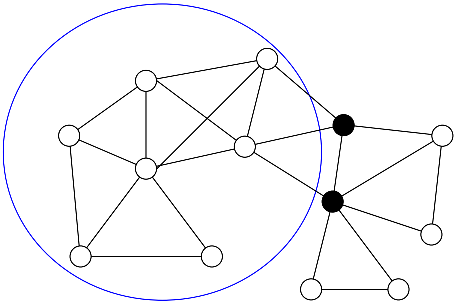

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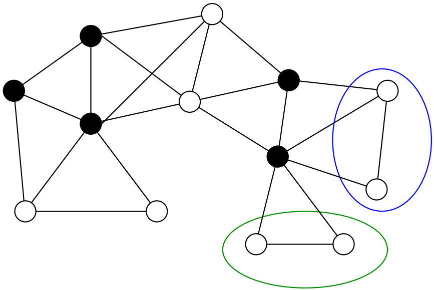

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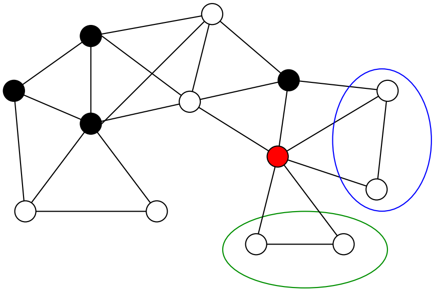

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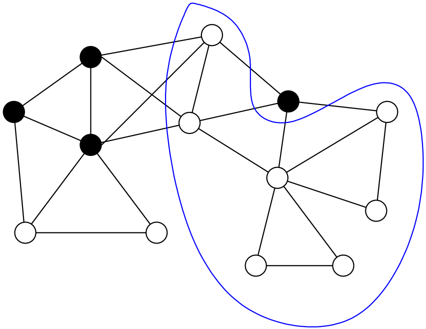


Region merging

Let $X \subset E$ and let A be a region for X.

- We say that *A can be merged* (for X) if there exists a region B, such that A and B can be merged.

Region merging and graphs

Remark

Based on region merging properties*, we can define* four classes of graphs*.*

- *Weak fusion graphs*
- *Fusion graphs*
- *Strong fusion graphs*
- *Perfect fusion graphs*

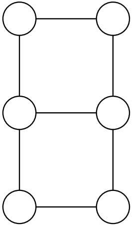
For clarity reasons, we will introduce only two of these four classes.

Fusion graph

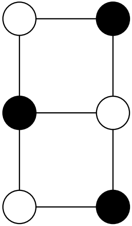
Definition

We say that (E, Γ) *is a* fusion graph *if for any subset of vertices* $X \subseteq E$*, any region for* X *can be merged.*

Fusion graph: example



Fusion graph: example



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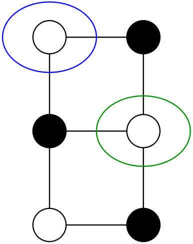

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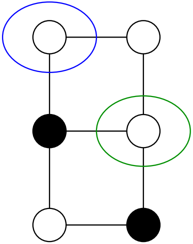

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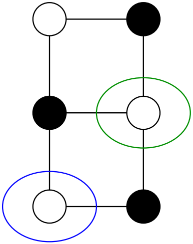

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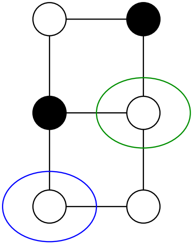

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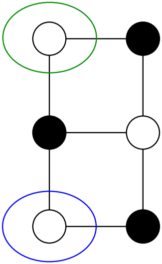

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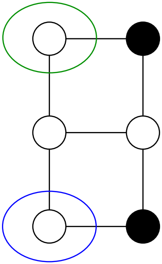

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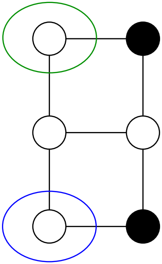


Problem

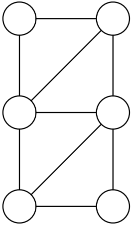
There exists neighboring regions that can not be merged through their common neighborhood.

Perfect fusion graphs

Definition

We say that (E,Γ) is a perfect fusion graph if, for any $X \subseteq E$, any two regions for X, which are neighbor, can be merged through their common neighborhood.

Perfect fusion graph, example



Fusion graphs: properties

Fusion graphs: properties

Property

Any perfect fusion graph is a fusion graph.
The converse is in general not true.

Characterization of Fusion Graphs

Theorem

A graph G is a fusion graph if and only if any non-trivial watershed in G is thin.

Characterizations of perfect fusion graphs

Theorem

The three following statements are equivalent:
i) (E, Γ) *is a perfect fusion graph;*

Characterizations of perfect fusion graphs

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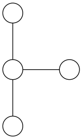
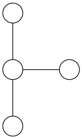
The three following statements are equivalent:
i) (E, Γ) *is a perfect fusion graph;*
ii) $G^{\blacktriangle}$ *is not a subgraph of* (E, Γ).



Characterizations of perfect fusion graphs

Theorem

The three following statements are equivalent:
i) (E, Γ) *is a perfect fusion graph;*
ii) $G^{\blacktriangle}$ *is not a subgraph of* (E, Γ).
iii) for any non-trivial watershed X *in* E, *any point in* X *is adjacent to exactly two regions for* X.



Problems

Problem

- *Given a grayscale image, how can we obtain an initial watershed set that can be used by further merging procedures?*

Problems

Problem

- *Given a grayscale image, how can we obtain an initial watershed set that can be used by further merging procedures?*
- *Topological grayscale watershed?*

Basic notion for vertex-weighted graphs

Let F be a map from E to $\mathbb{N}$. Let $k \in \mathbb{N}$.

- We denote by $F[k]$ *the set* $\{x \in E; F(x) \geq k\}$.
- A connected component of $\overline{F}[k]$ which does not contain a connected component of $\overline{F}[k-1]$ is a *(regional) minimum* of F.

W-destructible point

Let F be a map let $p \in E$ and let $k = F(p)$.

Definition

We say that p is W-destructible for F *if p is W-simple for $F[k]$.*

W-thinnings and topological watersheds

Let F and G be two maps.

Definition

- *We say that G is a W-thinning of F, if G may be derived from F by iteratively lowering W-destructible points by one.*

W-thinnings and topological watersheds

Let F and G be two maps.

Definition

- *We say that G is a W-thinning of F, if G may be derived from F by iteratively lowering W-destructible points by one.*
- *We say that G is a topological watershed of F if G is a W-thinning of F and if there is no W-destructible points for G.*

W-thinnings and topological watersheds

Let F and G be two maps.

Definition

- *We say that G is a W-thinning of F, if G may be derived from F by iteratively lowering W-destructible points by one.*
- *We say that G is a topological watershed of F if G is a W-thinning of F and if there is no W-destructible points for G.*

Definition

- *The set of all points which are not in a minimum of F, denoted by $\overline{M}(F) \subseteq E$ is the divide of F.*

Topological watersheds: example

3	5	5	5	10	10	10	10	15
3	5	30	30	30	30	30	15	15
3	5	30	20	20	20	30	15	15
40	40	40	20	20	20	40	40	40
10	10	40	20	20	20	40	10	10
5	5	40	40	20	40	40	10	5
1	5	10	15	20	15	10	5	0

Topological watersheds: example

3	5	5	5	10	10	10	10	15
3	5	30	30	30	30	30	15	15
3	5	30	20	20	20	30	15	15
40	40	40	20	20	20	40	40	40
10	10	40	20	20	20	40	10	10
5	5	40	40	20	40	40	10	5
1	5	10	15	20	15	10	5	0

3	3	3	3	3	3	3	3	3
3	3	30	30	30	30	30	3	3
3	3	30	1	20	0	30	3	3
30	30	30	1	20	0	30	30	30
1	1	1	1	20	0	0	0	0
1	1	1	1	20	0	0	0	0
1	1	1	1	20	0	0	0	0

Problem

Problem

- *Is the divide of a topological watershed a watershed set?*
- *Can we extend the thinness property of watershed set on fusion graphs to the grayscale case?*

Problem

- Counter-example on a fusion graph: the 8-connected graph.

0	9	0	8	0	9	0	7	0
0	9	9	8	9	9	0	7	0
0	0	9	8	9	7	7	7	0
0	0	9	9	9	6	6	6	6
0	0	9	8	9	7	7	7	0
0	9	9	8	9	9	0	7	0
0	9	0	8	0	9	0	7	0

Problem

- Counter-example on a fusion graph: the 8-connected graph.

I	9	A	8	B	9	C	7	D
I	9	9	8	9	9	C	7	D
I	I	9	8	9	7	7	7	D
I	I	9	9	9	6	6	6	6
I	I	9	8	9	7	7	7	E
I	9	9	8	9	9	F	7	E
I	9	H	8	G	9	F	7	E

Problem

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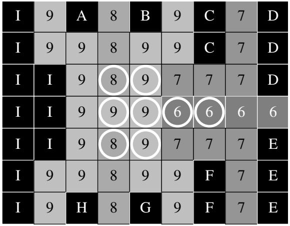

Problem

- Counter-example on a fusion graph: the 8-connected graph.

I	9	A	8	B	9	C	7	D
I	9	9	8	9	9	C	7	D
I	I	9	8	9	7	7	7	D
I	I	9	9	9	6	6	6	6
I	I	9	8	9	7	7	7	E
I	9	9	8	9	9	F	7	E
I	9	H	8	G	9	F	7	E

Problem

- *The divide of a topological watershed is not necessarily a watershed set and can be thick, even on fusion graphs.*

Problem

- *What about topological watersheds on perfect fusion graphs?*

M-cliff points

Let F be a map and let $x \in E$.

Definition

- *We say that* x is a cliff point (for F) *if* x *is W-simple for the divide of F (*i.e.*, if it is adjacent to a single minimum of* F*).*
- *We say that* x is M-cliff (for F) *if* x *is a cliff point with minimal altitude.*

M-cliff points

Let F be a map and let $x \in E$.

Definition

- *We say that* x is a cliff point (for F) *if* x *is W-simple for the divide of F* (i.e., *if it is adjacent to a single minimum of* F*).*
- *We say that* x is M-cliff (for F) *if* x *is a cliff point with minimal altitude.*

Property

If (E, Γ) *is a perfect fusion graph then any point M-cliff for* F *is W-destructible for* F*.*

Thin topological watershed

Theorem

On a perfect fusion graph, the divide of any topological watershed is:

- *a watershed set;*
- *a thin set.*

Algorithms for topological watersheds

Problem

The algorithms for topological watershed *are quasi-linear but* not linear*.*

- *Is there a faster (linear) algorithm on perfect fusion graphs?*

C-watersheds: definition

Let F and G be two maps.

Definition

- *We say that G is a* C-thinning of F *if G may be derived from F by iteratively lowering M-cliff point.*
- *We say that G is a* C-watershed of F *if G is a C-thinning of F and if there is no M-cliff point for G.*

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Remark

Let x be a M-cliff point.

- *If G is derived from F by lowering the value of x down to the altitude of the only minimum adjacent to x, then G is a C-thinning of F.*

C-watersheds: properties

Suppose that (E, Γ) is a perfect fusion graph.
Let F be a map and G be a C-watershed of F.

Property

- *G is a W-thinning of F.*
- *the divide of G is a watershed set.*
- *the divide of G is thin.*

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- On non-perfect fusion graphs, the previous properties are in general not true.

C-watersheds: algorithm

Data: a perfect fusion graph (E, Γ), a map F

Result: F

$L := \emptyset$; $K := \emptyset$;

Attribute distinct labels to all minima of F and label the points of $M(F)$ with the corresponding labels;

foreach $x \in E$ **do**

- **if** $x \in M(F)$ **then** $K := K \cup \{x\}$;
- **else if** x *is adjacent to* $M(F)$ **then** $L := L \cup \{x\}$; $K := K \cup \{x\}$;

while $L \neq \emptyset$ **do**

- $x :=$ an element with minimal altitude for F in L;
- $L := L \setminus \{x\}$;
- **if** x *is adjacent to exactly one minimum of* F **then**
  - Set $F[x]$ to the altitude of the only minimum of F adjacent to x;
  - Label x with the corresponding label;
  - **foreach** $y \in \Gamma^{\star}(x) \cap \overline{K}$ **do** $L := L \cup \{y\}$; $K := K \cup \{y\}$;

C-watershed: linear time algorithm

Property

- *C-watershed algorithm is monotone;*

C-watershed: linear time algorithm

Property

- *C-watershed algorithm is monotone;*
- *it runs in linear time with respect to the size of the input graph.*

Grids

Property

None of the usual grids is a perfect fusion graph.

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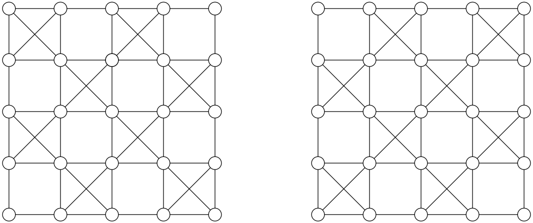
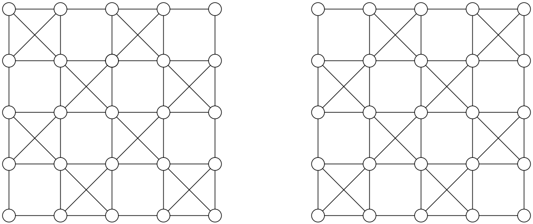


Grids

Property

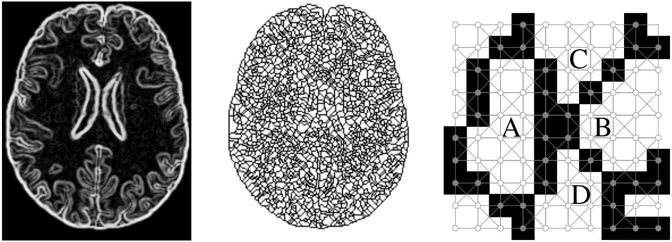
None of the usual grids is a perfect fusion graph.

- We introduce the perfect fusion grids.



Perfect fusion grids can be defined in dimension over $\mathbb{Z}^n$, for any integer n.

To conclude by an example



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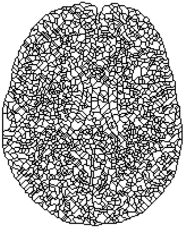

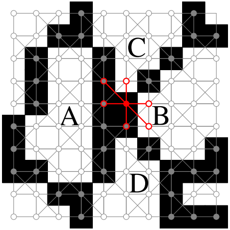

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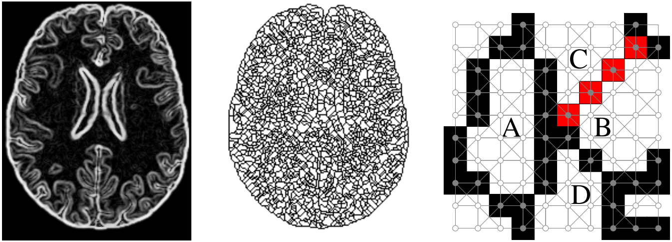

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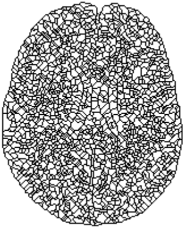
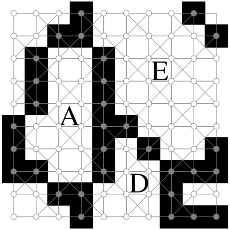




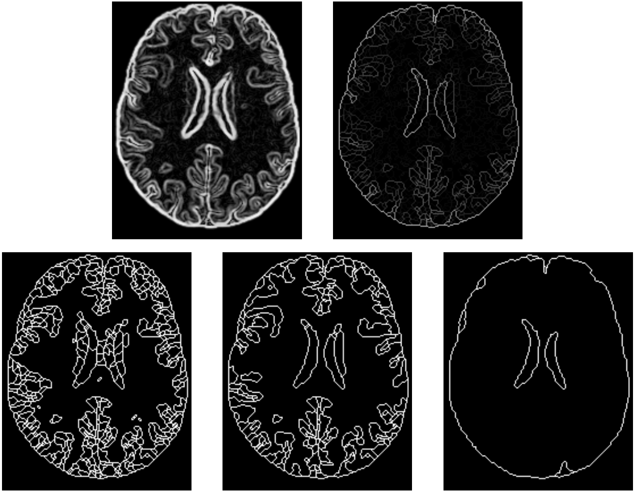
Conclusion

- Perfect fusion graphs: framework adapted for region merging methods based on grayscale watersheds
- Introduction of a simple linear-time algorithm to compute grayscale watersheds in this framework

Perspectives

- *Drop of water principle*:
  - A framework that guarantees the existence of such watersheds;
  - New simple and linear algorithms to compute those watersheds;
- *Region merging schemes*:
  - Links bewteen minimum spanning trees and watersheds;
  - Saliency and watershed hierachies.

Perspectives: saliancy on perfect fusion grids



Publications (Grayscale watersheds)

Theoretical foundations

G. Bertrand. *On topological watersheds.* vol. 22, n. 2-3, pp. 217-230 Journal of Mathematical Imaging and Vision, May 2005. (Special issue on Mathematical Morphology after 40 years)

Comparisons with flooding and the emergence paradigm

L. Najman, M. Couprie, and G. Bertrand. *Watersheds, mosaics and the emergence paradigm.* vol. 147, n. 2-3, pp. 301-324. Discrete Applied Mathematics, April 2005. (Special issue on DGCI)

Publications (Fusion graphs)

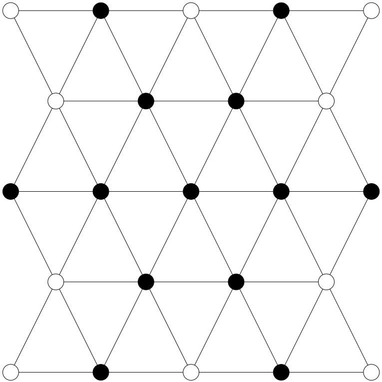
Fusion graphs

J. Cousty, G. Bertrand, M. Couprie, and L. Najman. *Fusion graphs: merging properties and watershed.* Computer Vision and Image Understanding, 2006. Submitted, Special Issue commemorating the career of Prof. Azriel Rosenfeld. Also in IGM2005-04.

Fusion graphs and grayscale watersheds

J. Cousty, M. Couprie, L. Najman and G. Bertrand. *Grayscale Watersheds on Perfect Fusion Graphs.* pp. 60-73. IWCIA 2006, LNCS 4040, proceedings, June 2006.

Grille hexagonale



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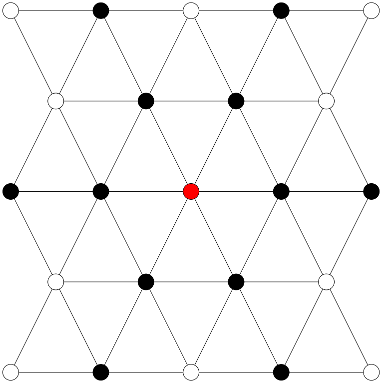
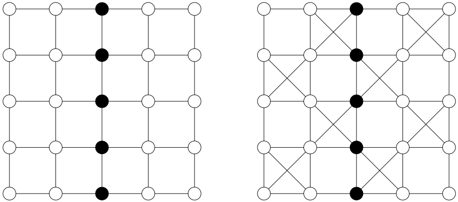


Division de régions



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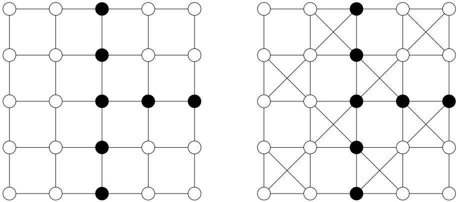

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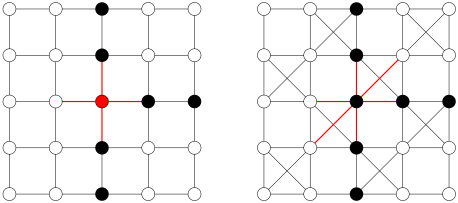

Division de régions

Property

Soit (E, Γ) un graphe de fusion parfait. Soit $X \subseteq E$, une LPE et A une région pour X. Si $Y \subseteq A$ est une LPE sur $(A, \Gamma \cap [A \times A])$ alors $X \cup Y$ est une LPE sur (E, Γ).

La propriété n'est pas vérifiée sur les graphes de fusion.