State Estimation of Dynamic Systems Using Viterbi Algorithm

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The problem of on-line estimation of time varying dynamic systems has been extensively treated in the literature. These systems are, in general, approximated by a model which is linear with respect to the disturbance and measurment white noise. The estimation of the states can be acheived by using the extended Kalman filter [1], [2].
In many applications, the state space model or the measurment model is non-linear. The linearization of these models gives estimates which are different from the actuel values.
In this paper, a new estimation algorithm is presented. Our approach is based on the Viterbi algorithm developped for decoding convolutional codes [3], [4], [5]. This recursive algorithm is optimal in the sense that it is the maximum-likelihood estimation of the entire received observation sequence [6]. It applies the dynamic programming principle to the detection of discrete-time finite state Markov processes with noisy observation.

We consider the general time-varying non-linear discrete models

<u>state model</u>

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{w_1}(k), \mathbf{u}(k), k)$$

<u>process output model</u>

$$\mathbf{y}(k) = \mathbf{g}(\mathbf{x}(k), \mathbf{w}_1(k), \mathbf{u}(k), k)$$

<u>observation model</u>

$$\mathbf{z}(k) = \mathbf{h}(\mathbf{y}(k), \mathbf{w}_2(k), \mathbf{u}(k), k)$$

where

$\mathbf{x}(\mathrm{k})$	state vector of dimension nx1
$\mathbf{u}(\mathrm{k})$	controllable inputs vector of dimension sx1
$\mathbf{w}_1(\mathrm{k})$	disturbance vector of dimension nx1
$\mathbf{y}(\mathrm{k})$	process outputs vector of dimension rx1
$\mathbf{w}_2(\mathrm{k})$	measurment errors vector of dimension rx1
$\mathbf{z}(\mathrm{k})$	observation vector of dimension rx1

for $k = 0, 1, 2, ..., K$

Note that parameter uncertainties are not considered and that the vector functions **f**, **g**, **h** may be linear time-varying or time-invariant vectors.
In order to use the Viterbi algorithm, the state space and perturbation spaces are quantized into non-overlapping subspaces. Hence, the viterbi algorithm models possible state sequences of the source Markov process as paths in the treillis, which represents the states at different time indices as nodes at different treillis stages and state transitions as arcs between nodes [4],[5]. A cost function is assigned to the nodes and transitions in the treillis. Hence the estimated state sequence is formed by the best path in the treillis.
A computer simulation study, using Matlab software, for some linear or non-linear time varying models will be conducted to evaluate the performance of the proposed estimation algorithm.

References

[1] Kalman, R.E., "A new approach to linear filtering and prediction problems", ASME Transactions, Journal of Basic Engineering, 82D,35-50 March 1960.
[2] Kalman, R.E., and Bucy, R.S., "New results in Linear Filtering and prediction", ASME Transactions, Journal of Basic Engineering, 83D,95-108 March 1961.
[3] Viterbi, A. J., "Error bounds for convolutional codes and an asymptotically optimum decoding algorithm", IEEE Trans. Inform. Theory, vol. IT-13,pp. 260-269, April 1967.
[4] Viterbi, A. J., and Omura, J. K., "Principles of Digital Communications and Coding", New York,McGraw-Hill, 1979.
[5] Forney, G. D., "The Viterbi algorithm", Proc.IEEE, pp. 268-278, March 1973.
[6] Forney, G. D., "Maximum-Likelihood sequence estimation of digital sequences in the presence of intersymbol inference", IEEE Trans. Inform. Theory,18, pp. 363-378, March 1972.