ADAPTIVE POLYNOMIAL PRE-DISTORTION FOR LINEARIZATION OF POWER AMPLIFIERS IN WIRELESS COMMUNICATIONS AND WLAN

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Abstract

This paper presents a new adaptive pre-distortion algorithm for linearization of Power Amplifiers and its application to non-constant envelope modulations such as QAM or OFDM. The pre-distortion system is polynomial. The criterion is minimization of a Mean Square Error criterion between the baseband-equivalent of the output of the real amplifier and the ideally amplified signal. The analytic expression of the gradient of the criterion has been calculated. A stochastic gradient algorithm is applied using this analytic expression. A special normalization of the coefficients of the polynomial predistortion has proposed to improve the speed of convergence. The method has been tested on a class AB power amplifier with a baseband signal corresponding to filtered QPSK and OFDM modulations.

1. Introduction

The design of very efficient Power Amplifiers (PA) is a crucial problem for mobile communications since it directly influences the autonomy of the mobile terminals. But efficient PA always present non-linearities that generate amplitude and phase distortions on the output signal. These distortions are characterized by AM-AM and AM-PM curves [1]. For digital mobile communications equipment's, they are the origin of spectral regrowth in adjacent channels and of deformation of the constellation (increasing of the EVM) increasing the demodulation error rate. These distortions depend on the envelope dynamic of the modulation. For constant envelope modulations such as GMSK, they are much less disturbing than for non-constant envelope modulations such as QAM or OFDM modulations.

Many techniques [2] have been proposed to compensate for these non-linearities such as feedback or feedforward analog techniques or adaptive pre-distortion techniques. We proposed recently [4] a new baseband polynomial pre-distortion technique, using an EVM criterion adapted to the case of EDGE modulation. In this paper, we generalize the approach to other types of modulations, and analyze the gradient and stochastic gradient properties for polynomial identification

2. Principle of the proposed method

Let us call $z(n)$ the complex envelope of the modulated signal and $z_I(n)$, $z_Q(n)$ the cartesian co-ordinates of the complex baseband signal $z(n)$. The polynomial pre-distortion function is called f. It is a complex polynomial function [3] of $|z(n)|^2$. For an input z(n), the output of the pre-distortion system is $z_p(n)= z(n)f(|z(n)|^2)$. With $a=|z(n)|^2$, the expression of f is:

$$f(a) = \sum_{k=1}^{K+1} f_k a^{K+1-k} \qquad (1)$$

The complex coefficients f_k are modified adaptively in order to minimize a criterion J. The proposed criterion J is a MSE criterion, where the error signal is the difference between the ideally amplified signal $G_0z(n)$ and the demodulated output $z_{p-a}(n)$ produced by the cascade of the predistorter and the Non Linear PA:

$$J = E(|e(n)|^2) = E\left(\left|z_{p\_a}(n) - G_0 z(n)\right|^2\right) \qquad (2)$$

Where G_0 is the desired gain of the amplifier (see section 5 B for more details on G_0).

It is possible to add an equalizing filter on each term of the difference $G_0z(n)$ and $z_{p-a}(n)$ [4].

Figure 1 represents the principle of the adaptive baseband pre-distortion method.

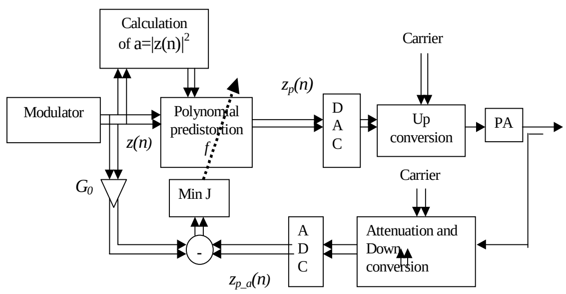

Figure 1: Diagram of the pre-distortion system

The simulations have been done using an equivalent baseband model that supposes that the output distorted signal is narrow-band compared to the carrier frequency.

The amplifier has also been simulated in baseband. The complex gain of the amplifier is a function of the squared envelope of its input signal a. Its value is deduced from the AM-AM and AM-PM measured curves.

3. Optimal predistorsion

the cascade of the predistorter and of baseband-equivalent of the power amplifier (noted PA) is represented figure 2.

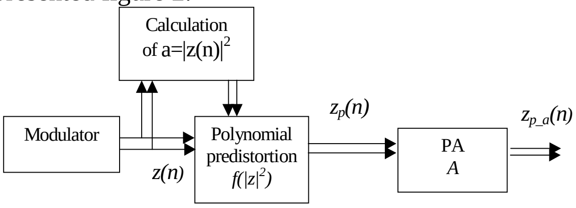

Figure 2: Cascade of distorter and baseband-equivalent of the power amplifier

In figure 2, the output of the predistoter is $z_p(n)$, and the output of the amplifier is $z_{p\_a}(n)$ with:

$$z_{p\_a}(n) = A(z_p) = e^{j\left(\arg(z_p)+\Phi_a\left(\left|z_p\right|^2\right)\right)}\left|A\left(\left|z_p\right|^2\right)\right|. \quad (3)$$

Where |A| and Φ_a are given by the AM-AM and AM-PM characteristics of the power amplifier.

The optimal solution for the predistorter gives an output z_a equal to $G_0 z$. The output of the predistorter is $z_p(n)= z(n)f(|z(n)|^2)$, so the optimal function f is :

$$f\left(\left|z\right|^2\right) = \frac{1}{z}A^{-1}(G_0 z). \quad (4)$$

Where A^{-1} represents the inverse function of A.

See section 5 for corresponding experiments and results.

4. LMS Algorithm for pre-distorsion adaptation

4.1. Stochastic gradient algorithm

According to the diagram of Fig. 1 and the choice described in **2.**, we minimize the following instantaneous criterion for adapting the pre-distorter (order p):

$$J(n) = \left|z_{p\_a}(n) - G_0 z(n)\right|^2 \quad (5)$$

We have used the LMS algorithm on that criterion, that is:

$\boldsymbol{f}(n) = \boldsymbol{f}(n-1) - \mu .* \nabla_f(n)$

(vectors are in boldface and ***u*.*v** stands for the term by term product of ***u*** and ***v***).

The gradient of $J(n)$ needs some steps of calculation. After these steps we obtain for the k^{th} coefficient f_k of the distorter:

$$\frac{\partial J(n)}{\partial f_k} = \left|z(n)\right|^2 a_n^{p+1-k} \left(\left(f(a_n)g(b_n) - G_0\right)\left(g(b_n) + b_n g'(b_n)\right)^* + \left(f(a_n)g(b_n) - G_0\right)^*\left(a_n\left(f(a_n)\right)^2 g'(b_n)\right)\right) \quad (6)$$

Where:

- a_n holds for $|z_n|^2$
- b_n holds for $a_n |f(a_n)|^2$.

Interpretation:

We observe that the correction increases with:

1. $f(a_n)g(b_n) - G_0$ that is the gain error.
2. a_n^{p+1-k} that is the partial derivative $\partial f(a)/\partial f_k$
3. $g(b_n) + b_n g'(b_n)$ that depends on the local gain and local slope of the gain of the amplifier.

4.2. Choice of the adaptation steps

The gradient value for parameter f_k depends on the values a^{p+1-k}, that is $|z|^{2(p+1-k)}$. In order to normalize the algorithm, we have used an adaptation step for parameter f_k that is proportional to: $1/E(|z|)^{3*(p+1-k)}$. This expression has been found experimentally and gives homogeneous convergence of all the polynomial components of ***f***.

In order to have a better insight in the convergence conditions of this type of algorithm we have studied the problem of polynomial identification with gradient and stochastic gradient algorithm for MSE criterion.

4.3. Polynomial Identification

We are looking for the coefficients of a polynome **g** generating the output y=x**g**(x^2) when submitted to the input x . We are proceeding according to the following scheme (Figure 3)

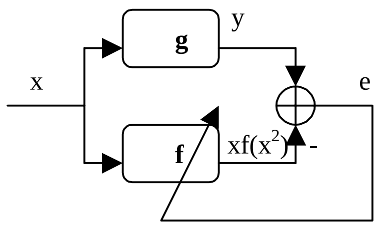

Figure 3: Polynomial identification system

The criterion chosen is the MSE between y (the reference signal) *and* $xf(x^2)$ produced by the polynomial form that we want to adjust. We can write $xf(x^2)$ under the form $\mathbf{p}^T\mathbf{f}$, where **p** is the vector of odd powers of x: $p_i = x^{2i+1}$ for i=0…order. We can write the criterion $J=\mathbf{f}^T E(\mathbf{p}\mathbf{p}^T)\mathbf{f}$ and its minimum is reached for $\nabla_\mathbf{f} J=\mathbf{0}$.

As $\nabla_\mathbf{f} J = 2\,E(\mathbf{p}(y - \mathbf{p}^T\mathbf{f}))$ the system is: $E(\mathbf{p}\mathbf{p}^T)\mathbf{f} = E(y\mathbf{p})$. The vector **g** satisfies this system and therefore the exact solution is $\mathbf{f}_{opt}=\mathbf{Q}^{-1}\mathbf{q}=\mathbf{g}$, with $\mathbf{Q}= E(\mathbf{p}\mathbf{p}^T)$ and $\mathbf{q}= E(y\mathbf{p})$. As the terms in **Q** present a huge dispersion, (for gaussian process of variance σ^2, $Q_{i,j} = E(x^{2(i+j+1)}) = (2(i+j+1))!/(i+j+1)!/2^{(i+j+1)}\,\sigma^{2(i+j+1)}$) we can introduce a first normalization by introducing a vector $\boldsymbol{\nu}$ such as $\nu_i = f_i * E(x^{2i})$. Then we obtain the system $\boldsymbol{\nu}_{opt}=\mathbf{R}^{-1}\mathbf{q}$ where $R_{i,j} = Q_{i,j} / E(x^{2j})$.

Gradient algorithm: As the criterion is a quadratic form in $\boldsymbol{\nu}$ (or **f**) we can reach the unique minimum by using a gradient algorithm: $\boldsymbol{\nu}^{(n+1)} = \boldsymbol{\nu}^{(n)} + \mu .* (\mathbf{q} - \mathbf{R}\boldsymbol{\nu}^{(n)})$.

A normalized algorithm is then obtained if the step of adaptation μ_i is chosen such as the correction terms for ν_i are of same order of magnitude.

We have considered $\mu_i = \mu_0 / E(x^{2(i+1)})$. The resultant updating equation is therefore: $\boldsymbol{\nu}^{(n+1)} = \boldsymbol{\nu}^{(n)} + \mu_0 .* (\mathbf{p} - \mathbf{P}\boldsymbol{\nu}^{(n)})$ with **P** and **p** given by: $P_{i,j} = E(x^{2(i+j+1)}) / E(x^{2(i+1)}) / E(x^{2j})$,

$p_i=E(yx^{(2i+1)})/E(x^{2(i+1)})$. Figure 4 gives the results of this gradient algorithm for **g**=[2 50 10000] and x is a gaussian signal normalized by its maximum absolute value.

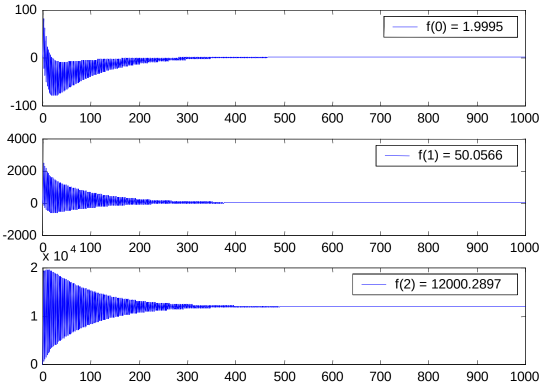

Figure 4: Convergence of polynomial coefficients with normalized gradient algorithm.

Stochastic gradient algorithm: We have then tried a normalized stochastic algorithm determined by the updating equation: $\mathbf{v}^{(n+1)} = \mathbf{v}^{(n)} + \mu_0.*(\mathbf{s}(n)-\mathbf{S}(n)\mathbf{v}^{(n)})$ with **S**(n) given by: $S_{i,j}(n)= x_n^{2(i+j+1)}/E(x^{2(i+1)})/E(x^{2j})$, and **s**(n) given by $s_i(n) = y_n x_n^{(2i+1)}/E(x^{2(i+1)})$. Fig 5 gives the results for the same **g**. This LMS algorithm must be driven with a very small adaptation step which leads to very slow convergence (~100000 iterations instead of ~1000 for the true gradient with optimal step).

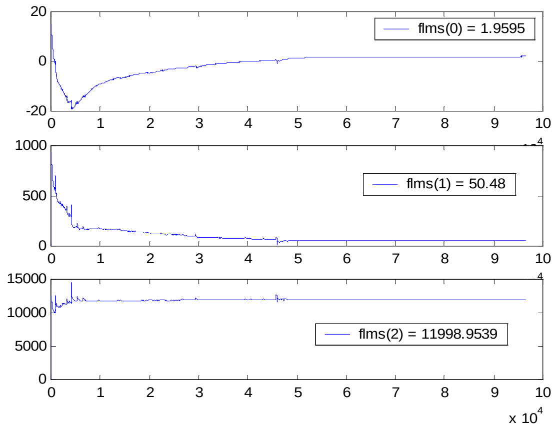

Figure 5: Convergence of polynomial coefficients with normalized stochastic gradient algorithm.

Conclusion: we didn't reach our initial goal which was to justify the normalized step which was found to work better for our pre-distortion problem. However we have analysed the gradient type algorithm for the polynomial identification problem (in this case our previous approach is not valuable and does not leads to correct results).

5. Experiments and results

5.1. Characteristic of tested Modulations

We have tested the predistortion algorithms with 2 kinds of non-constant envelope modulations:

– QPSK with a root-raised cosine shaping filter of rolloff equal to 0.22,

– OFDM modulation.

These 2 modulations are well representative of new standards for mobile communications of 3rd or 4th generation and WLAN.

An important parameter to evaluate the influences of the non-linearities of a power amplifier on a modulation is the ratio of the peak power to the mean power of the envelope signal. This ratio will be called MMR (Max Mean Ratio).

For the chosen filtered QPSK modulation, MMR is equal to 5 dB. When the number of carrier is high enough, the envelope of OFDM modulation presents a Rayleigh density of probability. MMR is equal to 10 dB. Figure 6 gives the histogram of filtered QPSK envelope and OFDM envelope.

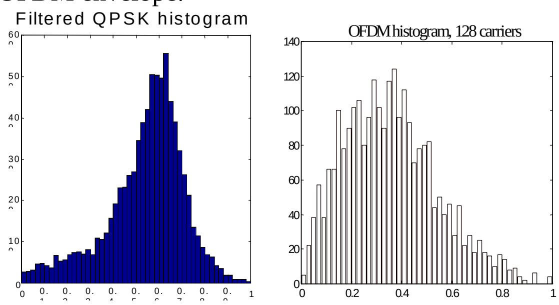

Figure 6: amplitude histograms of filtered QPSK qnf OFDM modulations

5.2. Amplifier

We have made simulations of our algorithm for an amplifier of class AB (weakly non-linear) developed by Motorola and given as an example in the CAD software HPADS. The amplifier is designed for PCS applications and uses MOSFET technology.

As we work fully in base-band for the moment, we have considered an approximation of the complex gain characteristics of that amplifier by a 9-order polynomial. Fig. 7 presents the gain in amplitude and phase of the amplifier and its polynomial approximation.

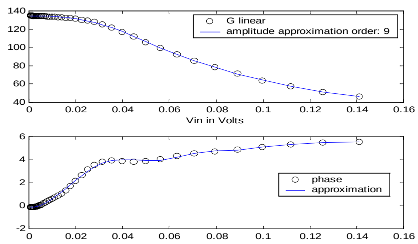

Figure 7: Complex Gain (amplitude and phase) of the amplifier, and its Polynomial Approximation.

5.3. Peak Back-Off and effects of non linear amplifier

The power amplifier can generate a maximum output power, noted P_{max}. In linearization experiments, one's tries to reach a certain percentage ρ of P_{max}. The PBO or Peak Back-Off is defined as $-10 \log_{10}(\rho)$.

For example, ρ=95% corresponds to a PBO of 0.22 dB, and ρ=80% corresponds to a PBO of 0.97dB.

For the maximum predistorted input power, the output power of the amplifier is equal to $\rho\ P_{max}$.

We have fixed the ideal gain C from the PBO. This ideal gain is defined as the ration $Vout_{max}/Vin_{max}$, with $Pout_{max}=\rho\ P_{max}$. This gain is slightly smaller than the gain in the linear area of the amplifier.

Figure 8 shows the filtered QPSK constellation at emitter, without and with non linear power amplifier.

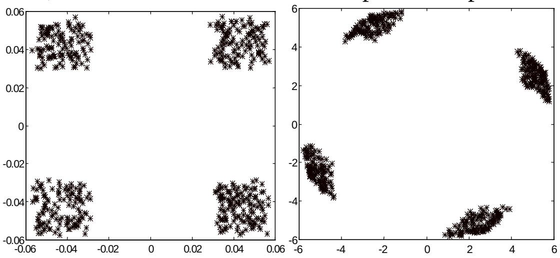

Figure 8: constellation at the output of the emitter without and with linear power amplifer.

Figures 10 and 11 show the effects of the NLA on the power spectral densities of the QPSK and OFDM modulations.

5.4. Fixed predistortion

We have first calculated a fixed predistortion. This fixed predistortion is obtained by fitting a 3^{th} order polynomial to the inverse curve of the complex gain of the amplifier. Figure 9 shows the inverse curve of the gain and its approximation by a polynomial curve.

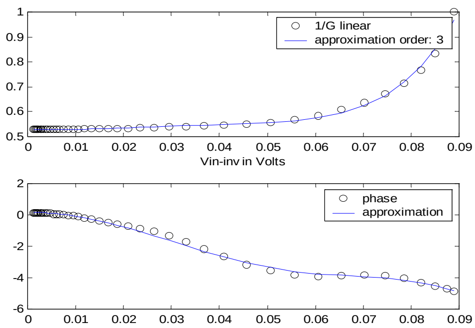

Figure 9: Inverse complex Gain and its Polynomial Approximation.

The results obtained for the power spectral densities, with the fixed pre-distortion, are given Figure 10 and 11.

5.5. Adaptive predistortion

We have then adapted a 3^{th} order pre-distortion system with a neutral initialization ($f(|z^2|)=1$).

The adaptation has been done for different values of the PBO. After a few thousands iterations we obtain a system which gives the following results expressed in terms of EVM and power spectral densities performances.

EVM minimization: for QPSK signals the EVM falls from .089 to .0032.

Spectral consequences: The efficiency pre-distortion is also evaluated by the reduction of spectral re-growth. Figures 10 and 11 illustrate the power spectral densities (in baseband) for the 4 signals resulting from amplification of QPSK and OFDM signals respectively :

- Output of the NLA
- Output of an ideal (linear) amplifier with constant gain corresponding to the linear range of the amplifier
- Output of the fixed pre-distortion plus NLA.
- Output of the adaptive pre-distortion plus NLA.

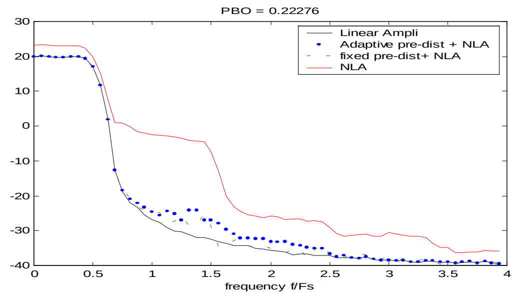

Figure 10: Power Spectral Densities before and after pre-distortion

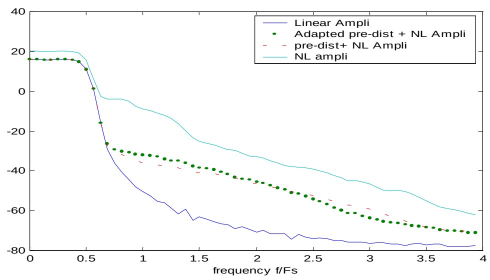

Figure 11: Power Spectral Densities before and after pre-distortion

We can notice that the shape of the power spectral density (psd) has been drastically improved whereas it was not the direct criterion. The adaptive pre-distortion performs almost as good as the fixed one.

6. Conclusion

We have developed an adaptive baseband polynomial pre-distortion for a non-linear amplifier, based on a Mean Square Error criterion. We have observed that it is important for convergence to use an adaptation step adapted to the degree of each polynomial coefficients.

References

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