From post-distortion to pre-distortion for power amplifiers linearization

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***Abstract*— This paper presents a new method of digital adaptive pre-distortion for linearization of Power Amplifiers (PA). The method is derived from a post-distortion approach which identifies the PA inverse function. This approach leads to the minimization of a quadratic function of the polynomial coefficients in the case of a polynomial predistorter form and a least square criterion. We have compared our solution to a method previously proposed by Ghaderi that was also based on the transformation of a post-distortion into a pre-distortion system. We have tested our predistorter (along with a baseband adaptation of Ghaderi's one) on OFDM hiperlan signals. Both methods significantly reduced the signal distortion and the spectral regrowth. Our less complex approach proved to be even better for small Peak Back Off values.**

***Index Terms*— Pre-distortion, linearization, power amplifiers.**

I. INTRODUCTION

EFFICIENT power amplifiers (PA) present non-linearities generating amplitude and phase distortions on the PA output signal. These distortions create spectral regrowth in adjacent channels and deformation of the signal constellation. They highly depend on the dynamics of the input amplitude. In order to achieve better spectral efficiency, the emerging systems of mobile communications and local area networks use non-constant envelope modulations and therefore are more sensitive to these distortions.

Many techniques have been proposed to compensate for non-linearities [1]. This paper deals with digital baseband pre-distortion methods. It uses the formal frame of equivalent baseband models. The principle of pre-distortion is to distort the PA input signal by an additional device called a predistorter (PD) whose characteristics are the inverse of those of the amplifier. If the PA is considered as a memoryless system, its distortions can be characterized by the AM/AM and AM/PM characteristics, which give respectively the output power and phase as a function of the input power [1]. The designer has to choose the Peak Back Off (PBO) value (the difference in dB between the maximal desired output power and the saturation power). The resulting gain G_0 of the cascade PD + PA is the gain of the PA for this maximum output power. If z is a complex envelope at the PA input, the low pass filtered complex envelope at the PA output is: $\mathcal{A}(z) = AM\left(|z|^2\right)e^{j\left(arg(z)+PM\left(|z|^2\right)\right)}$ where $AM(\left(|z|^2\right)$ and $PM\left(|z|^2\right)$ are the AM/AM and AM/PM characteristics.

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II. COMMON ARCHITECTURE FOR LINEARIZATION WITH PRE-DISTORTION

The usual approach is illustrated in Fig. 1. The predistorter is set to minimize a given criterion comparing the attenuated output of the amplifier and the original signal. The predistorter

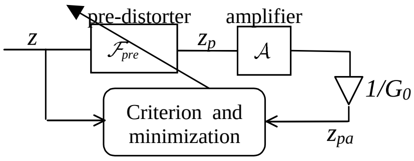

Fig. 1. Common Architecture for the pre-distortion.

can be implemented by a lookup table or by an analytical function such as a polynomial function. It generally acts as a complex gain depending on the input magnitude. For a given input z, the output z_p of the predistorter is:

$$z_p = \mathcal{F}_{pre}(z) = z\, G_{pre}\left(|z|\right). \tag{1}$$

The ideal predistorter function can be implicitly defined by: $\mathcal{A}(\mathcal{F}_{pre}(z)) = G_0 z$. So the ideal solution for $\mathcal{F}_{pre}$ verifies $\mathcal{F}_{pre}(z) = \mathcal{A}^{-1}\left(G_0 z\right)$.

In the architecture of Fig. 1, the observations that are used to obtain the ideal solution are z and $\mathcal{A}(\mathcal{F}_{pre}(z))$. As $\mathcal{A}$ is a non linear function, the solution $\mathcal{F}_{pre}$ cannot be written explicitly from these observations and has to be derived by classical iterative optimization techniques. In a previous paper [2], we tested this architecture using a polynomial form for $\mathcal{F}_{pre}(z)$ and a mean square error criterion with a stochastic gradient algorithm to adapt the coefficients. The algorithm converged easily for WCDMA signals or $3\pi/8$ 8PSK Edge signals, but difficulties occured for OFDM signals in the case of small PBO.

III. PROPOSED PRE-DISTORTION ARCHITECTURE DERIVED FROM A POST-DISTORTION APPROACH

If we consider the symmetric problem of post-distortion (Fig. 2), the ideal postdistorter $\mathcal{F}_{post}$ should give: $\mathcal{F}_{post}\left(\frac{1}{G_0}\mathcal{A}(z)\right) = z$, or equivalently: $\mathcal{F}_{post}(z_a) = \mathcal{A}^{-1}\left(G_0 z_a\right)$, with $z_a = \frac{1}{G_0}\mathcal{A}(z)$. [1] Note that the optimal expressions of $\mathcal{F}_{post}$ and $\mathcal{F}_{pre}$ are identical. The important consequence of this is that we can calculate

[1] z_a represents the output of the amplifier multiplied by $\frac{1}{G_0}$. More generally, the order of indices added to the variable z indicates the successive systems encountered by z. For example z_{pap} denotes the output of the cascade of **p**redistorter-**a**mplifier coupled with the scaled **p**ostdistorter.

a pre-distortion system from a post-distortion diagram. The main interest is that the solution for post-distortion is directly expressed from the observations z and z_a.

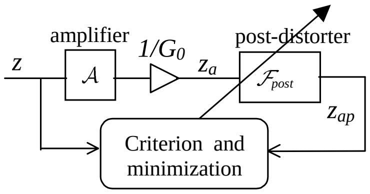

Fig. 2. post-distortion scheme

It should be noted that this approach consists in identifying the PA inverse function $\mathcal{A}^{-1}$ using the input and output of the PA.

We have proposed one solution to make the system adaptive. We continuously identify the PA inverse function $\mathcal{A}^{-1}$ and translate it in an adaptive way into a pre-distortion function. We have named this method the "translation method".

IV. Adaptivity of the translation method

Let $\mathcal{F}_{pre}^{(n)}$ be the current predistorter and $z_p(n)$, $z_{pa}(n)$ the input signal and the output signal of the power amplifier respectively. The proposed method (Fig. 3) uses these two signals to calculate a new estimate $\mathcal{F}_{post}^{(n)}$ of the inverse function of the PA. We apply this solution as the new predistorter: $\mathcal{F}_{pre}^{(n+1)}(z) = \mathcal{F}_{post}^{(n)}(z)$. If the algorithm converges, $\mathcal{F}_{pre}$ and

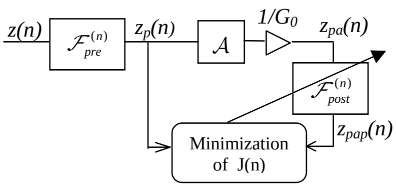

Fig. 3. Adaptive Translation method

$\mathcal{F}_{post}$ have the same limit $\mathcal{F}_{pre}^{(\infty)}$, which implies that:

$$\mathcal{A}\left(\mathcal{F}_{pre}^{(\infty)}(z)\right) = G_0 z.$$

This last equation shows that the linearization has been achieved.

This approach has similarities to Ghaderi's method [3]. We compare the two approaches in section V.

A. Recursive Least Square (RLS) Implementation for polynomial predistorters

We have implemented this approach with a polynomial form for the pre and post distorter. The postdistorter is applied to z_{pa} according to:

$$z_{pap} = \mathcal{F}_{post}(z_{pa}) = z_{pa}\, G_{post}(|z_{pa}|) = z_{pa} \sum_{i=0}^{K} f_i^{post} |z_{pa}|^{2i} \tag{2}$$

The criterion minimized at sample n is:

$$J(n) = \sum_{l=1}^{n} \lambda^{n-l} |e(l)|^2 = \sum_{l=1}^{n} \lambda^{n-l} |z_p(l) - z_{pap}(l)|^2, \tag{3}$$

where:

- λ is a forgetting factor,
- $z_{pap}(l) = z_{pa}(l) \boldsymbol{f}(n)^T \boldsymbol{a}(l)$, and:
  - $\boldsymbol{f}(n)^T = \left(f_0^{post}(n)\ f_1^{post}(n) \dots f_K^{post}(n)\right)$,
  - $\boldsymbol{a}(l)^T = \left(1\ |z_{pa}(l)|^2 \dots |z_{pa}(l)|^{2K}\right)$.

The criterion $J(n)$ is a quadratic function of the coefficients in the vector $\boldsymbol{f}(n)$. Its unique minimum at sample n corresponds to: $\boldsymbol{f}(n) = \boldsymbol{P}^{-1}(n)\boldsymbol{q}(n)$ with:

- $\boldsymbol{P}(n) = \sum_{l=1}^{n} \lambda^{n-l} |z_{pa}(l)|^2 \boldsymbol{a}(l)\boldsymbol{a}(l)^T$ and:
- $\boldsymbol{q}(n) = \sum_{l=1}^{n} \lambda^{n-l} z_p(l) z_{pa}^*(l) \boldsymbol{a}(l)$.

A RLS algorithm was used with the following initializations: initial predistorter set to $\boldsymbol{f}(0) = (1\ 0\ \dots 0)^T$ and $\boldsymbol{P}(0) = 10^{-5}\boldsymbol{I}$. At each step the vector $\boldsymbol{f}(n)$ is applied as a new PD vector for sample $n+1$.

V. Comparison of the translation method with Ghaderi's approach

The translation method presents similarities with the approach proposed by Ghaderi [3]. Ghaderi uses a polynomial predistorter that is applied on analog intermediate frequency (IF) quadrature signals. His design necessitates complex analog circuitry. A digital RLS algorithm is also used to adapt the coefficients of the predistorter. This necessitates analog to digital converters on the quadrature down-converted signals.

Apart from the IF or baseband implementation, there are several other fundamental differences between our translation method and Ghaderi's approach. We briefly describe here the Ghaderi's approach, highlighting the differences with our technique.

1) In Ghaderi's approach, the quadrature demodulator (QDM) after the PA uses the predistorter input signal as a local oscillator. If $z(t) = r_i(t)e^{j\theta_i(t)}$ and $z_{pa}(t) = r_o(t)e^{j\theta_o(t)}$ are respectively the complex envelopes of the modulated input and output signals, the complex envelope of the signal at the output of the QDM is: $r_i(t)r_o(t)e^{j(\theta_o(t)-\theta_i(t))}$.
2) This signal is divided by $r_i(t)$ (obtained by an envelope detector) and a constant phase Φ is added to $(\theta_o(t) - \theta_i(t))$. The resulting quadrature signals are: $\begin{cases} x_{ia}(t) = r_o(t)cos\left(\theta_o(t) - \theta_i(t) + \Phi\right) \\ x_{qa}(t) = r_o(t)sin\left(\theta_o(t) - \theta_i(t) + \Phi\right) \end{cases}$. The phase Φ helps to improve the convergence speed of the adaptive algorithm presented below.
3) At the n^{th} iteration, the postdistorsion is made of two quadrature polynomial gain functions $Y_{I,n}$ and $Y_{Q,n}$ of the two variables $x_{ia,n}$ and $x_{qa,n}$:
   $Y_{I,n}(x_{ia,n}, x_{qa,n}) = \sum_{k=1}^{M} a_{ii,k,n} x_{ia,n}^{k-1} + \sum_{k=1}^{M} a_{iq,k,n} x_{qa,n}^{k-1}$,
   $Y_{Q,n}(x_{ia,n}, x_{qa,n}) = \sum_{k=1}^{M} a_{qi,k,n} x_{ia,n}^{k-1} + \sum_{k=1}^{M} a_{qq,k,n} x_{qa,n}^{k-1}$.
   These expressions should be compared to our complex postdistorter gain G_{post} which is a function of the single variable $|z_{pa}|$ (eq. 2).

4) In Ghaderi's approach, the predistorter gain polynomials at the n^{th} iteration are expressed in term of powers of the input amplitude signal as:
$G_{I,n}(r_i(n)) = \sum_{k=1}^{M} \alpha_{i,k,n}(r_i(n))^{k-1}$,
$G_{Q,n}(r_i(n)) = \sum_{k=1}^{M} \alpha_{q,k,n}(r_i(n))^{k-1}$.
This is equivalent to our expression using complex coefficients $f_k^{pre}(n)$:
$G_{pre}^{n}(r_i(n)) = \sum_{k=0}^{K} f_k^{pre}(n)(r_i(n))^{2k}$.
5) The error functions used by Ghaderi for the data sampled at the n^{th} iteration are defined as:
$e_I(n) = r_i(n)G_{I,n}\left(r_i(n)\right) - r_o(n)Y_{I,n}\left(x_{ia,n}, x_{qa,n}\right)$,
$e_Q(n) = r_i(n)G_{Q,n}\left(r_i(n)\right) - r_o(n)Y_{Q,n}\left(x_{ia,n}, x_{qa,n}\right)$.
In contrast, we used a complex error function defined as:
$e(n) = z(n)G_{pre}^{n}(r_i(n)) - z_{pa}(n)G_{post}^{n}(r_o(n))$
6) In Ghaderi's approach, the relationship between the predistorter and postdistorter coefficients at the n^{th} iteration is (for $k = 1, 2, \cdots, M$):
$\alpha_{i,k,n} = (cos(\Phi))^{k-1}a_{ii,k,n-1} + (sin(\Phi))^{k-1}a_{iq,k,n-1}$,
$\alpha_{q,k,n} = (cos(\Phi))^{k-1}a_{qi,k,n-1} + (sin(\Phi))^{k-1}a_{qq,k,n-1}$.
However, we have the straightforward relation :
$f_k^{pre}(n) = f_k^{post}(n-1)$, for $k = 0, 1, \cdots, K$.
7) Ghaderi used 2 real cost functions J_I and J_Q:
$J_I(n) = \sum_{l=1}^{n} e_I^2(l)$, $J_Q(n) = \sum_{l=1}^{n} e_Q^2(l)$.
These are quadratic functions of the coefficients $(a_{ii,k,n}, a_{iq,k,n})$ and $(a_{qi,k,n}, a_{qq,k,n})$ respectively. The solution is obtained by a RLS algorithm with a re-initialization every ten iterations in order to reduce the significance of data in the distant past.
We used a single cost function J with a forgetting factor (eq. 3).
From points 3, 4, 5, we see that phase and amplitude are processed in the same way in the predistorter, in the postdistorter and in the error function in our approach, which is not the case in Ghaderi's method.

VI. Experiments and results

We have tested the adaptive translation method and compared it with the method proposed by Ghaderi. We used the Saleh model for the PA. This model does not correspond to the application of personal or cellular radio-communications but it is widely used as a reference. The baseband input signal is an OFDM signal with QPSK symbols on data sub-carriers. We have tested the method with PD polynomial order K=3 (and the equivalent for Ghaderi's method).

The PBO was set to 0.22 dB, which is much more severe than the cases tested in [3] (1 to 4 dB). The sampling frequency was set to 16 F_S (F_S=symbol rate). The forgetting factor was set to : $\lambda = 0.8$.

We have observed (Fig. 4) the instantaneous errors $|G_0 z(n)| - |G_0 z_{pa}(n)|$ and $\arg(G_0 z(n)) - \arg(G_0 z_{pa}(n))$. Figure 5 shows the power spectral densities (computed after convergence) of $G_0 z(n)$ (ideally amplified signal), $G_0 z_{pa}(n)$ (output of the PA with pre-distortion) and $G_0 z_a(n)$ (output of the PA without pre-distortion). We can see that:

- Our algorithm shows very fast convergence (after a few samples) whereas Ghaderi's algorithm locks onto the right module and phase only after a short (a few hundred samples) but random duration.
- After the convergence, Ghaderi's method is still slightly less accurate than our approach but both methods give significant reduction of error and spectral distortions.

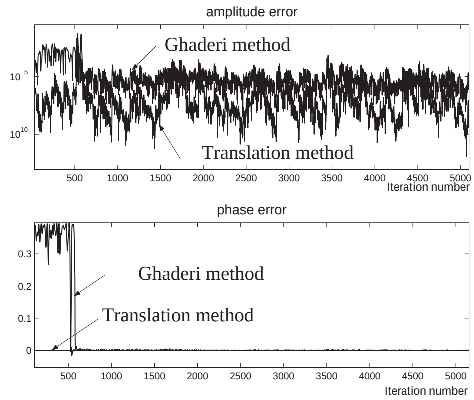

Fig. 4. Instantaneous errors (amplitude and phase) between input and output signals

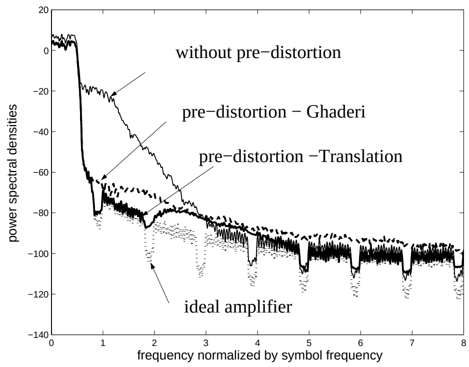

Fig. 5. Spectral densities of signals

VII. Conclusions

In this paper, a method for PA linearization using pre-distortion derived from a postdistorsion approach has been presented and compared to a method previously proposed by Ghaderi. Both methods lead to similar results for a least square error criterion but Ghaderi's method requires a longer convergence time. Our translation approach is less complex and leads to better results.

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