Monte-Carlo Estimation of Time Mismatch Effect in a OFDM EER Architecture

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***Abstract* – New 5 GHz Wireless Local Area Networks standards uses OFDM modulation in order to increase data rate transfer. OFDM transmitter needs linearization technique due to non-linearities of the power amplification operation. EER architecture can be used to solve this problem while keeping a high efficiency. However several sources of imperfections lower the quality of the signal. Time mismatch has especially a great impact on EVM and spectral re-growths. This paper presents a Monte Carlo study of envelope/phase delay influence on the OFDM signal. The Autocorrelation is estimated considering the OFDM signal as complex Gaussian.**

I. INTRODUCTION

New 3rd Generation standards such as Hiperlan2 or IEEE 802.11a uses OFDM (Orthogonal Frequency Division Multiplex) at 5 GHz. The advantages are a high data rate transfer and robustness in multi-paths environment. Each of the sub-carriers (64) uses a QAM modulation scheme. The high disadvantage of OFDM is that the envelope of the emitted signal exhibits a large amplitude range. Consequently the power is un-constant. This results in distortions caused by non-linearities in the radio-frequency transmitter (especially the power amplifier). Linearization methods are necessary. EER (Envelope Elimination and Restoration) introduced by Kahn in 1957 [1] is a solution to linearize the transmitter while keeping high efficiency. EER is based on the decomposition of the emitted signal in a magnitude signal (envelope varying) and a phase signal (constant power). Each signal is amplified separately. As shown on Fig. 1, recombination of the envelope and phase information is done by supply modulation of the high efficiency RF power amplifier (PA)

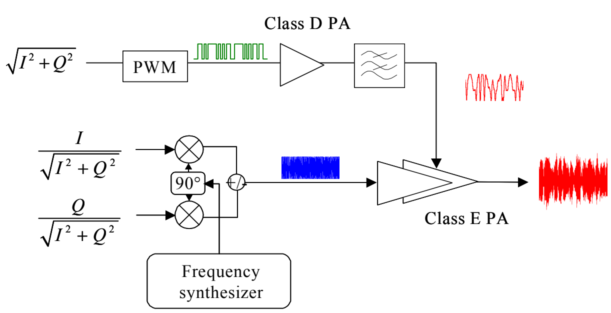

Fig. 1: EER architecture principle

In this architecture, several sources of imperfections lower the quality of the transmission [2]. The major impact is due to time mismatch Δ between envelope and phase signals at the recombination. This is caused by different operations on each of the two path. Consequences are noise on the information and phase rotation [2] of the sub-carriers constellation. A statistical Monte Carlo analysis enables a characterization of delay influence on the emitted signal. The design of such architecture needs to precisely quantify the spectral effects of the previous defaults. A possible approach would be to study statistically the power spectrum obtained at the output of a simulated system, while varying values of the potential defaults. In fact, the impact of these defaults can be directly analysed using simulated signal generated with the same statistical properties as those of the potential outputs of the system. Hence, the results are obtained without resorting on a complete simulation of the whole system, that is interesting as far as computational load and simulation duration are concerned. It is also interesting in the fact that it focuses on the actual defaults under concern and not on other 'hidden' defaults on the simulation process.

II. THE SHIFTED OFDM SIGNAL

Because of time mismatch between envelope and phase components, the actual emitted signal is formed as the product of the envelope with a delayed version of the phase factor: if $x(t)=\rho(t)e^{j\phi(t)}$ is the original OFDM signal, then the 'shifted' (distorted) OFDM signal is $x_\Delta(t)=\rho(t)e^{j\phi(t-\Delta)}$. In the OFDM context, the emitted signal *x(t)* can be modelled as a complex circular Gaussian process, when the number of sub-carrier Nsub is high enough (typically greater than 30), because of the central limit theorem. Such a signal is completely characterized by its mean and autocorrelation function, which is here analytically known (as a function of the emission filter). But what can be said about the statistical properties of the shifted signal? As a non-linear transform of a gaussian process, $x_\Delta(t)$ may not be a gaussian process itself. Since the envelope $\rho(t)$ and phase $\phi(t)$ derive from a correlated process, there might be some statistical dependence between $\rho(t)$ and $\phi(t-\Delta)$. In the case of a complex circular gaussian process (e.g. a band-pass signal with a symmetric spectrum around its central frequency and I/Q parts with gaussian amplitude, such as our OFDM signal) it is well known that the envelope and phase are independent and respectively distributed according to Rayleigh and uniform distributions, The case of delayed envelope and phase is not given in the standard literature and seems to be little or not known. In fact, it appears that $\rho(t)$ and $\phi(t-\Delta)$ are also independent variables with Rayleigh and uniform distributions, for all Δ, so that the distribution of $x_\Delta(t)$ is also gaussian. The proof is as follows: at time *t* and *t* + τ the joint distribution of ρ and ϕ is

$$p(\rho_1,\phi_1,\rho_2,\phi_2)=\frac{\rho_1\rho_2}{(2\pi\sigma^2)^2(1-R^2)}\exp\left(-\frac{\rho_1^2+\rho_2^2+R\rho_1\rho_2\cos(\phi_1-\phi_2)}{2\sigma^2(1-R^2)}\right),$$

where σ^2 is the variance of *x(t)* and $R^2=|R_{XX}(\tau)|^2/2\sigma^2$ if

$R_{XX}(\tau)$ denotes the autocorrelation function of *x(t)*. The joint distribution between $\rho(t)$ and $\phi(t-\Delta)$ is obtained by marginalization of $p(\rho_1,\phi_1,\rho_2,\phi_2)$ with respect to ϕ_1 and ρ_2, that is $p(\rho_1,\phi_2)=\iint p(\rho_1,\phi_1,\rho_2,\phi_2)d\phi_1 d\rho_2$. Beginning by the integration versus ϕ_1 and using the integral representation of modified Bessel function, we obtain

$$p(\rho_1,\phi_2)=\frac{\rho_1}{(2\pi\sigma^2)^2(1-R^2)}\exp\left(-\frac{\rho_1^2}{2\sigma^2(1-R^2)}\right)$$
$$\times\int_0^{+\infty}\rho_2\exp\left(-\frac{\rho_2^2}{2\sigma^2(1-R^2)}\right)I_0\left(\frac{R\rho_1\rho_2}{2\sigma^2(1-R^2)}\right)d\rho_2$$

and then using the integration formula [11-4-29] in [5], together the relation between regular and modified Bessel functions, we arrive at

$$p(\rho_1,\phi_2)=\frac{1}{2\pi}\times\frac{\rho_1}{(2\pi\sigma^2)^2}\exp\left(-\frac{\rho_1^2}{2\sigma^2}\right)=p(\rho_1)\times p(\phi_2),$$

that does not depend on the correlation coeffient anymore and shows that $\rho(t)$ and $\phi(t-\Delta)$ are independent with Rayleigh and uniform distributions respectively. Therefore, the amplitude of $x_\Delta(t)$ is gaussian distributed. But this does not mean that $x_\Delta(t)$ is a gaussian process. For that, all joint distributions between several instants should only depend on a correlation function. What we are ultimately interested in is the emitted spectrum. By Wiener-Kintchine theorem, it is the Fourier transform of the autocorrelation function $R_{x_\Delta x_\Delta}(\tau)$ of the shifted OFDM signal $x_\Delta(t)$ This writes

$$R_{x_\Delta x_\Delta}(\tau)=E\left[x_\Delta(t)x_\Delta^*(t-\tau)\right],$$ that is

$$R_{x_\Delta x_\Delta}(\tau) = E\left[\rho(t).e^{j\varphi(t-\Delta)}\rho(t-\tau)e^{-j\varphi(t-\Delta-\tau)}\right] \quad (1)$$

where $E[.]$ is the statistical average operator with the underlying probability distribution

$$p(\rho(t),\varphi(t-\Delta),\rho(t-\tau),\varphi(t-\Delta-\tau)). \quad (2)$$

Note that even in the case Δ=0 it can be shown that

$$p(\rho(t),\varphi(t-\Delta),\rho(t-\tau),\varphi(t-\Delta-\tau))\neq p(\rho(t),\rho(t-\tau))\times p(\varphi(t),\varphi(t-\tau))$$

That indicates that the processes ρ(t) and φ(t) are not independent, the latter being only true for the statistic at one instant. The last distribution (1) can only be expressed by marginalizing the 8 variables joint distribution

$$p(\rho(t),\varphi(t),\rho(t-\Delta),\varphi(t-\Delta),\rho(t-\tau),\varphi(t-\tau),\rho(t-\Delta-\tau),\varphi(t-\Delta-\tau))$$

with respect to $\varphi(t),\rho(t-\Delta),\varphi(t-\tau)$ and $\rho(t-\Delta-\tau)$. This is a formidable, if not impossible task, and have to resort to another technique so as to evaluate the spectrum of the `shifted' OFDM signal.

III. Simulations

For Hiperlan2 (Nsub = 64) we model the OFDM signal as a Gaussian signal with known autocorrelation Rxx(τ). The Autocorrelation may be estimated as the time average from the output $x_\Delta(t)$ of a simulated system :

$$\hat{R}x_\Delta x_\Delta(\tau)=\frac{1}{N}\sum_{i=1}^{N}\rho(i-\tau).e^{-j\varphi(i-\Delta-\tau)}.\rho(i).e^{j\varphi(i-\Delta)}$$

The idea here is to estimate the autocorrelation (1) by ensemble average. This means that $Rx_\Delta x_\Delta(\tau)$ is computed as the mean of function of K independent realisations of 4 random variables $\{\rho_1, \varphi_2, \rho_4, \varphi_3\}$ with the correct statistics:

$$\hat{R}x_\Delta x_\Delta(\tau)=\frac{1}{K}\sum_{k=1}^{K}\rho_1(k).e^{-j\varphi_2(k)}.\rho_4(k).e^{j\varphi_3(k)} \quad (3)$$

Hence, the random variables $\{\rho_1, \varphi_2, \rho_4, \varphi_3\}$ in (3) must have the same joint distribution as the variables $\{\rho_{(t)}, \varphi_{(t-\Delta)}, \rho_{(t-\tau)}$ and $\varphi_{(t-\tau-\Delta)}\}$ in (2). Equivalently, we shall generate a vector $\underline{W}=\{w_1,w_2,w_3,w_4\}$ with same joint distribution as a vector of samples of our OFDM signal $\underline{X}$:

$$\underline{X}=\{x(t),x(t-\Delta),x(t-\Delta-\tau),x(t-\tau)\}$$
$$\underline{X}=\left\{\rho(t)e^{j\varphi(t)},\rho(t-\Delta)e^{j\varphi(t-\Delta)},\rho(t-\Delta-\tau)e^{j\varphi(t-\Delta-\tau)},\rho(t-\tau)e^{j\varphi(t-\tau)}\right\}$$

As $\underline{X}$, the samples of our OFDM signal, is a Gaussian vector, it suffices to impose that $\underline{W}$ and $\underline{X}$ have the same correlation matrix. This correlation matrix *is analytically known* (since it depends on the known autocorrelation Rxx(τ)), and it is easy to generate $\underline{W}$ as a simple transform of a random gaussian vector $\underline{G}$ with uncorrelated components. The correlation matrix is given by: (where * denotes the hermitian transpose)

$$\underline{\underline{R}}_{\Delta,\tau}=E[\underline{X}\underline{X}^*]=\begin{pmatrix} Rxx(0) & Rxx(\Delta) & Rxx(\Delta+\tau) & Rxx(\tau) \\ Rxx(-\Delta) & Rxx(0) & Rxx(\tau) & Rxx(-\Delta+\tau) \\ Rxx(-\Delta-\tau) & Rxx(-\tau) & Rxx(0) & Rxx(-\Delta) \\ Rxx(-\tau) & Rxx(\Delta-\tau) & Rxx(\Delta) & Rxx(0) \end{pmatrix}$$

The key of the simulation process is to remark that given any square root $\underline{\underline{C}}$ of $\underline{\underline{R}}$ (i.e. a cholesky factor or a matrix of eigenvectors) such that $\underline{\underline{R}}=\underline{\underline{C}}\underline{\underline{C}}^*$, we have:

$$E[\underline{W}.\underline{W}^*]=E[\underline{\underline{C}}\underline{G}\underline{G}^*\underline{\underline{C}}^*]=E[\underline{\underline{C}}\underline{\underline{C}}^*]=\underline{\underline{R}}_{\Delta,\tau}$$

with $\underline{W}=\underline{\underline{C}}\underline{G}$ and $\underline{G}$ a random zero mean Gaussian vector with uncorrelated components. So, for a given Δ, the simulation consist in:

for some values of τ , τ_n, with n=1..N
for k=1 to K number of realizations
- *Generate a Gaussian vector $\underline{G}$ with uncorrelated components.*
- *Compute $\underline{W}_{k,n}=\underline{\underline{C}}\underline{G}$*
- *$A_{k,n}=|W_{k,n}(1)|$, $B_{k,n}=e^{(j.phase(W_{k,n}(2)))}$, $C_{k,n}=e^{(-j.phase(W_{k,n}(3)))}$, and $D_{k,n}=|W_{k,n}(4)|$. Set $T_{k,n}=A_{k,n}B_{k,n}C_{k,n}D_{k,n}$.*
end for.
- *Compute :* $$\hat{R}_{x_\Delta x_\Delta}(\tau_n)=\hat{E}[T]=\frac{1}{K}\sum_{k=1}^{K}A_{k,n}B_{k,n}C_{k,n}D_{k,n}$$

end for.

And finally compute the spectrum by Fourier transform. Results of autocorrelation simulation are reported on Fig. 2 where the delay is varied from 0 to 40% of the symbol time (20 nano-sec in Hipertlan2 case) considering a 20 MHz OFDM signal with a root raised cosine shape filter (roll-off = 0.5). Results show that delay causes small variations on the autocorrelation response. Resulting spectrums, for the same time mismatch, show spectral re-growths. When compared to Hiperlan2 spectral limit, a delay of 5 nsec is too high to fulfil standard requirements. Simulations are compared with HP-ADS Hiperlan2 ones (Fig. 4). Results we reported in [2] showed a limit of 3 nsec.

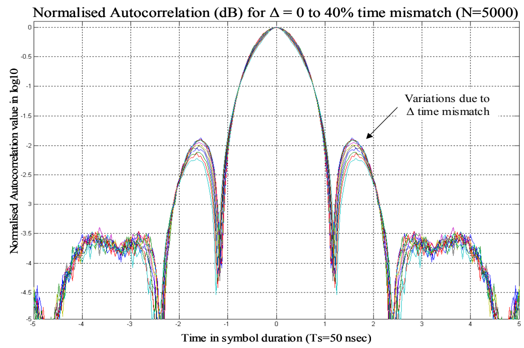

Fig. 2: Autocorrelation response from simulations

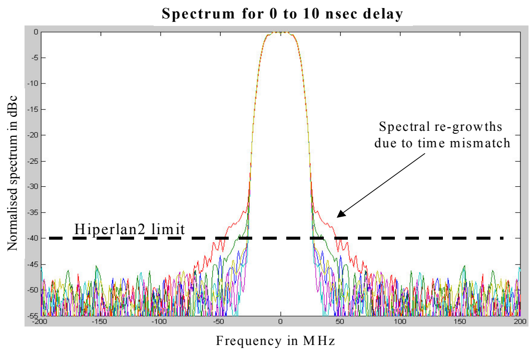

Fig. 3: Spectrum from simulated autocorrelations

The difference is explained by difference in spectrum calculation (windowing, averaging…) in HP-ADS, non-ideal Gaussian behaviour of Hiperlan2 simulated signal, or defaults in the simulation model. The accuracy can be improved by increasing K (10000 here). Confidence intervals can be computed using Student tests.

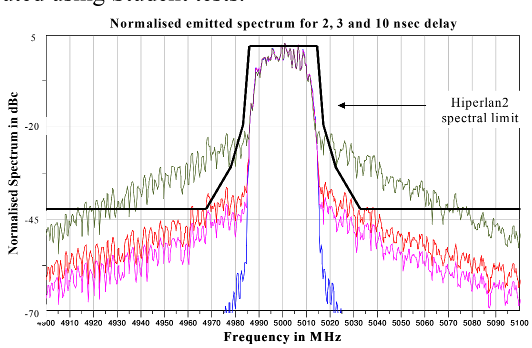

Fig. 4: Simulated spectrum with HP-ADS

The measure of maximum spectral re-growths is plotted on Fig.5 as a function of time delay. The spectral re-growths have an exponential behaviour with Δ, which confirms the great impact of the delay on the quality of emitted signal.

IV. CONFIDENCE INTERVAL OF THE ESTIMATION

The quality of autocorrelation estimation can be assessed using confidence intervals, that give the interval around the current estimate where should lie the true value, at a given level of probability. It is well known that the normalized variable

$$\frac{\sqrt{K}\left(\hat{R}_{X_\Delta X_\Delta}(\tau)-R_{X_\Delta X_\Delta}(\tau)\right)}{S_n} \qquad (3)$$

with S_n to be the unbiaised variance estimate, follows a (K-1) Student law.

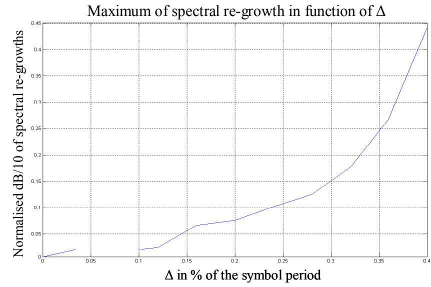

Fig. 5 : Maximum of spectral re-growths in function of Δ

A confidence interval is typically defined at 95% probability. In our case , it writes

$$P\left(-t_{\alpha/2} \le \frac{\sqrt{K}\left(\hat{R}_{X_\Delta X_\Delta}(\tau)-R_{X_\Delta X_\Delta}(\tau)\right)}{S_n} \le t_{\alpha/2}\right)=95\%$$

with P the probability function of (K-1) Student law. For high K, this law T(K-1) converges to a normalized Gaussian N(0,1), and one can substitute the Gaussian statistics to the Student's, for say K>50. The 95% confidence interval of the autocorrelation is then defined as:

$$P\left(\underbrace{\hat{R}_{X_\Delta X_\Delta}(\tau)-\frac{S_n}{\sqrt{K}}t_{\alpha/2}}_{\text{bottom limit}} \le R_{X_\Delta X_\Delta}(\tau) \le \underbrace{\hat{R}_{X_\Delta X_\Delta}(\tau)+\frac{S_n}{\sqrt{K}}t_{\alpha/2}}_{\text{upper limit}}\right)=95\%$$

For 95 % and K > 1000, $t_{\alpha/2}$ is set to 1.96 (the value given by T(K-1) is 1.598), and it remains to specify the value of S_n.
An unbiased estimator of the variance is given by

$$S_{K,\Delta,\tau} = \frac{1}{K-1}\sum_{k=1}^{k=K}\left(T_{k,\Delta,\tau}-E[T_{k,\Delta,\tau}]\right)^2 = \frac{1}{K-1}\sum_{k=1}^{k=K}\left(T_{k,\Delta,\tau}-\hat{R}_{X_\Delta X_\Delta}(\tau)\right)^2$$

The estimation of the standard deviation for any given Δ and τ is reported on Fig. 6.

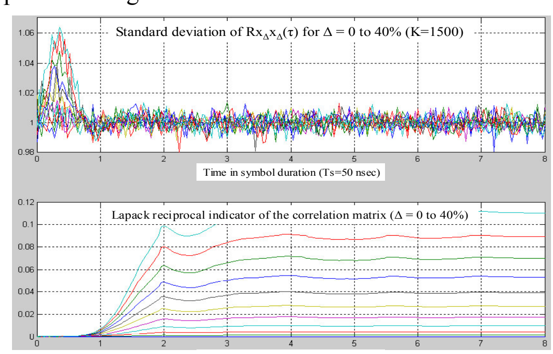

Fig. 6 : Standard deviation and Lapack indicator for K=1500

Values of the standard deviation at τ = 0 and infinity are constant whatever Δ. The Lapack indicator reveals that $\underline{R}$ is badly conditioned for low values of τ, especially when Δ and τ are in the same range. But in fact the variance of $\hat{R}_{x_\Delta x_\Delta}(\tau_n)$ can be derived analytically. Indeed, because the vectors $T_{k,n}$ are independent

$$\hat{R}_{x_\Delta x_\Delta}(\tau_n) = \frac{1}{K}\sum_{k=1}^{K} T_{k,n}$$

the variance of $\hat{R}_{x_\Delta x_\Delta}(\tau_n)$ is simply $Var[T_n]$/K and it remains to find $Var[T_n] = Var[A_{k,n}B_{k,n}C_{k,n}D_{k,n}]$.

Let $Y = A_{k,n}B_{k,n}$ and $Z = C_{k,n}D_{k,n}$. Now, let Z = $Z_1 + Z_2$, where Z_1 is perfectly correlated with variable Y, i.e. $Z_1 = \sqrt{\alpha}Y$, and Z_2 is uncorrelated with Y. Then $E[YZ_1^*] = \sqrt{\alpha}E[|Y|^2]$ and $E[YZ_2^*] = 0$. The variance can be expressed : $Var[T_n] = E[|YZ^*|^2]-|E[YZ^*]|^2$. Expanding, we obtain: $E[YZ^*] = \sqrt{\alpha}E[|Y|^2] + E[Y]E[Z_2^*]$

and $$E\left[|YZ^*|^2\right] = \alpha E[|Y|^4] + E[|Y|^2]E[|Z_2|^2] + 2E[|Y|^2 Z_1]E[Z_2]$$

From our discussion in section II, Y and Z are complex circular Gaussian process with zero mean, independent real and imaginary parts with variances equal to σ^2. It remains finally:

$$Var[T_n] = \alpha E[|Y|^4] + (1-2\alpha)\left|E[|Y|^2]\right|^2$$

because Y and Z are centered, and lastly $Var[T_n] = 4\sigma^4$, since $E[|Y|^4] = 8\sigma^4$, and $E[|Y|^2] = 2\sigma^2$. Hence it appears that <u>the variance is constant</u> for any τ or Δ. That corresponds to Fig. 6 results, where $\sigma^2 = ½$.

Finally the confidence interval can be plotted. At any given Δ, the evolution of the 95% estimation is reported on Fig. 7.

95% Confidence interval for $Rx_2x_2(\tau)$

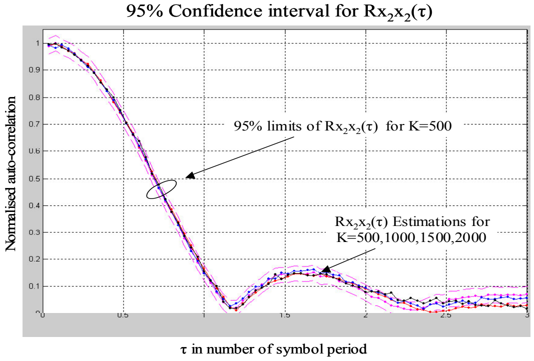

Fig. 7 : 95% confidence interval for Δ = 8% and K = 500 to 2000

The Confidence interval width is proportional to the variance of the autocorrelation estimation with

$$\widehat{VAR\left(\widehat{R_{x_\Delta x_\Delta}}(\tau)\right)} = \widehat{VAR\left(\frac{1}{K}\sum_{n=1}^{n=K} T_{n,\Delta,\tau}\right)} = \frac{4.\sigma^4}{K}$$

That can be confirmed by comparing the proportional width of the confidence interval relative to the estimate of the autocorrelation for different K. Results of table 1 show the importance of keeping K greater than 1000 and reveal a degradation of our estimation for increasing τ and time mismatch Δ. The last step is to take into account theses results in order to derive a confidence interval for the spectrum estimate. Since the estimates $\hat{R}_{x_\Delta x_\Delta}(\tau_n)$ are computed independently for all τ_n, the errors are independent, but with the same variance. Hence, the estimate of the autocorrelation can be understood as the true value corrupted with a Gaussian white noise:

$$\hat{R}_{x_\Delta x_\Delta}(\tau_n) = R_{x_\Delta x_\Delta}(\tau_n) + \xi(\tau_n).$$

Table 1 : Confidence interval width for different K

Evolution of confidence interval width in function of K, Δ, and τ		Values		Average Value	
		τ = 0 to 40%	τ = 40 to 80%	τ = 0 to Ts	τ = 1.4 to 2.Ts
K=500	Δ = 0	< 4.2%	< 8.9%	7%	34%
	Δ = 12%	< 4.3%	< 8.9%	7%	34.2%
	Δ = 36%	< 4.8 %	< 9%	7%	49%
K=1000	Δ = 0	< 3%	< 6.8%	5.2%	15%
	Δ = 12%	< 3.2 %	< 6.8%	5.2%	16.3%
	Δ = 36%	< 3.4%	< 7%	5.3%	19.8%
K=4000	Δ = 0	< 1.5%	< 3.2%	2.5%	11.6%
	Δ = 12%	< 1.5%	< 3.2%	2.5%	12%
	Δ = 36%	< 1.7%	< 3.3%	2.5%	15.7%

Therefore, the estimated spectrum writes

$$\hat{S}_{x_\Delta x_\Delta}(f) = S_{x_\Delta x_\Delta}(f) + \xi(f),$$

where $\xi(f)$, the Fourier transform of the white Gaussian noise, is a white Gaussian noise with variance $4\sigma^4$/K, and a confidence interval can be plotted accordingly (see Fig. 8).

Normalised Spectrum for Δ = 8% time mismatch (K=1500)

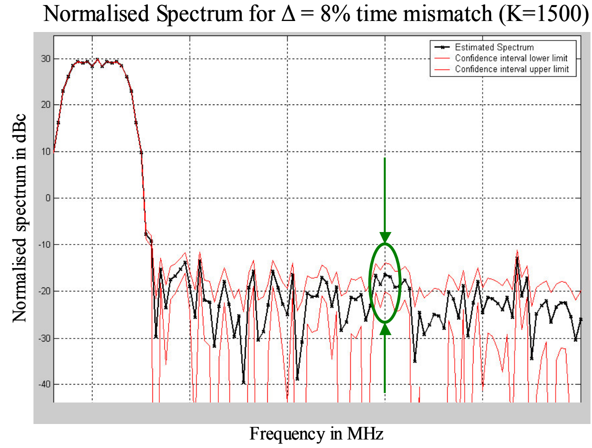

Fig. 8 : Emitted spectrum and confidence interval for Δ=8% and K=1500

V. Conclusion

An estimation of envelope/phase time mismatch influence on an OFDM signal was presented. This is particularly important in a sensibility analysis of EER architecture. Results showed a good agreement between simulated Monte Carlo study results and HP-ADS Hiperlan2 simulation ones. Characterizing the autocorrelation of the envelope delayed OFDM signal is possible. With simulated spectrums values, the delay imperfection can be analysed, and the impact of delay imperfection can be investigated. Accuracy of our Monte Carlo model is discussed and quantified in terms of confidence intervals.

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