A New Image Segmentation Framework : Power Watersheds

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Abstract

We summarize the work presented in [4] to ICCV09 that expresses a common energy function corresponding to the graph cuts, random walker, shortest path and optimum spanning forest optimization algorithms for seeded image segmentation. We also propose a new family of segmentation algorithms producing optimal spanning forests which we term **power watersheds**. *Placing the watershed algorithm in this energy minimization framework also opens new possibilities for using unary terms in traditional watershed segmentation and using watersheds to optimize more general models of use in applications beyond image segmentation. We illustrate the framework with a novel application to unseeded segmentation.*

Index Terms— Optimization, Image Segmentation, Optimal Spanning Forest

1 Introduction

The modern variations on interactive segmentation algorithms are primarily built on top of a small set of core algorithms — graph cuts (GC), random walker (RW) and shortest paths forests(SPF). Recently these three algorithms were all placed into a common framework that allows them to be seen as instances of a more general seeded segmentation algorithm with different choices of a parameter acting as an exponent on the differences between neighboring nodes [7]. In addition to these algorithms, the ubiquitous seeded watershed segmentation algorithm [2] shares a similar seeding interface but only recently was a connection made between the watershed algorithm and graph cuts [1]. We summarize here how this connection between watersheds and graph cuts can be used to further generalize the seeded segmentation framework of [7] such that watersheds, graph cuts, random walker and shortest paths may all be seen as special cases of a single general seeded segmentation algorithm.

2 A seeded image segmentation framework

A graph consists of a pair $G = (V, E)$ with vertices $v \in V$ and edges $e \in E \subseteq V \times V$ with cardinalities $n = |V|$ and $m = |E|$. An edge, e, spanning two vertices, v_i and v_j , is denoted by e_{ij}. In image processing applications, each pixel is typically associated with a node and the nodes are connected locally via a 4 or 8-connected lattice. A weighted graph assigns a (typically non-negative and real) value to each edge called a weight. The weight of an edge e_{ij} is denoted by $w(e_{ij})$ or w_{ij}. We also denote w_{Fi} and w_{Bi} as the unary weights penalizing foreground and background affinity at node v_i. In the context of segmentation and clustering applications, the weights encode nodal affinity such that nodes connected by an edge with high weight are considered to be strongly connected and edges with a low weight represent nearly disconnected nodes. One common choice for generating weights from image intensities is to set

$$w_{ij} = \exp(-\beta (I_i - I_j)^2), \tag{1}$$

where I_i is the image intensity at node (pixel) v_i.

Given foreground F and background B seeds, our model for producing segmentation s is given by

$$\begin{aligned} \min_x \sum_{e_{ij} \in E} & w_{ij}^p |x_i - x_j|^q + \\ & \sum_{v_i} w_{Fi}^p x_i^q + \sum_{v_i} w_{Bi}^p (x_i - 1)^q, \\ & \text{s.t. } x(F) = 1, \ \ x(B) = 0, \\ & s_i = 1 \text{ if } x_i \geq \frac{1}{2}, 0 \text{ if } x_i < \frac{1}{2}. \end{aligned} \tag{2}$$

It was shown in [5] that if the seeds are the maxima of the weighting function, Maximum Spanning Forest(MSF) are equivalent to watersheds. Recently, [1] showed that when $q = 1$ (graph cuts) and $p \rightarrow \infty$ then the minimum of (2) is given by a maximum spanning forest algorithm. Said differently, it was shown in [1] that as the power of the weights increases to infinity, then the graph cuts algorithm produces a segmentation corresponding to a maximum spanning forest. Interpreted from the standpoint of the Gaussian weighting function in (1), it is clear that we may associate $\beta = p$ to understand that the watershed equivalence comes from operating the weighting function in a particular parameter range. An important insight from this connection is that *above some value of β we can replace the expensive max-flow computation with an efficient maximal spanning forest computation*. We state in the next section that this statement is true for any of the algorithms corresponding to different q.

When p is a small finite value, then the various values of q may be interpreted respectively as the graph cuts ($q =$

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q \ p	0	finite	∞
1	Reduction to seeds	Graph cuts [7]	Maximum Spanning Forest (Watershed [5]) [1]
2	ℓ_2-norm Voronoi	Random walker [7]	Power watershed $q = 2$
∞	ℓ_1-norm Voronoi	ℓ_1-norm Voronoi	Shortest Path Forest [7]

Table 1: Our generalized scheme for image segmentation includes several popular segmentation algorithms as special cases of the parameters p and q. The power watersheds are previously unknown in the literature, but may all be optimized efficiently with a maximal spanning forest calculation.

1), random walker ($q = 2$) and shortest paths (geodesics) ($q = \infty$) algorithms which form the underpinning for many of the advanced image segmentation methods in the literature. When $p \to \infty$ and $q = 1$, then [1] showed that (2) may be interpreted as an MSF algorithm. However, by raising $p \to \infty$ and varying the power q we obtain a previously unexplored family of segmentation models which we refer to as **power watersheds**. An important advantage of power watersheds with varying q is that the main computational burden of these algorithms depends on an MSF computation, which is extremely efficient [3]. In the next section we study the case $p \to \infty$ there exists a value of p after which any of the algorithms (regardless of q) may be computed via an MSF. Table 1 gives a reference for the different algorithms generated by various value of p and q.

We give in [4] an algorithm to minimize (2) for any value of $q > 1$ when $p \to \infty$.

3 The case $p \to \infty$, q finite

We generalize the link between GC and MSF established by Alene *et al.* [1] by proving that GC, RW, and generally all q-cuts converge to MSF as p tends to infinity.

We define a q-cut by the set of edges having their vertices in two different connected components of the segmentation s. A MSF cut is the set of edges that links two different connected components of a maximum spanning forest. If M is a subgraph of G and if each weight w is unique, then any q-cut relative to M for $[w]^p$ when $p \to \infty$ is a MSF cut relative to M for w.

The labeling solution x of any q-cut relative to M for $[w]^p$ when $p \to \infty$ and all the weights are different is binary, as illustrated on Fig. 1.

In the case of an arbitrary set of weights (some weights can be equal), any q-cut relative to M for $[w]^p$ when $p \to \infty$ is a MSF cut relative to M for w when M is the set of all maxima of the image. This is due to the fact that by adding the edges to the forest by decreasing weight order, we only encounter plateaus (connected set of edges of same weight) in order. A method for forcing any set of markers to be the only maxima of an image is to apply a reconstruction [6].

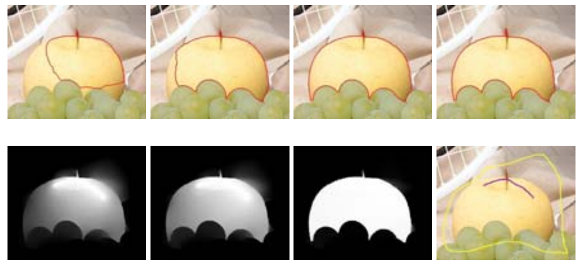


Figure 1: Illustration of progressive convergence to the power watershed result as $p \to \infty$, using $q = 2$. Top row: Segmentation results with $p = 1$, $p = 8$, $p = 25$ and the power watershed. Bottom row: Corresponding potentials for $p = 1$, $p = 8$, $p = 25$ and the input seeds.

4 Seeded image segmentation

We compared the algorithms of our framework using a 2D vision database of images with ground truth and seeds from the "grabcut" project (http://research.microsoft.com/en-us/um/cambridge/projects/visionimagevideoediting/segmentation/grabcut.htm). Example segmentations for five algorithms of our framework (graph cuts, random walker, shortest paths, maximum spanning forests (watersheds) and powerwatersheds with $q = 2$) with the original Grabcut database seeds are shown in [4].

The seeds provided by the Grabcut database being equidistant from the ground truth boundary, to remove any bias from this seed placement on our comparative results, we produced an additional set of seeds by significantly eroding the original foreground seeds.

When segmenting with the first seeding strategy (the seeds contained in the Grabcut database), all the algorithms are comparable. Experiments on the second set of seeds show that power watersheds and GCs yield better results than the other algorithms. The RW and the SPF algorithms show good results for the first set of seeds because these two algorithms do well when the seeds are placed roughly equidistant from the desired boundary [7], as they are in the seeds provided with the Grabcut database. In contrast, the power watershed performed very well under both seeding strategies, showing a strong robustness to both seed quantity and location. More details can be found in [4].

5 Example of unseeded segmentation

We now present an application of the framework to unseeded segmentation. This application is not presented in [4]. The unary terms in (2) are treated as binary terms connected to phantom seeds v_F and v_B, i.e.,

$$\sum_{v_i} w_{Fi}^p (x_i - 0)^q + \sum_{v_i} w_{Bi}^p (x_i - 1)^q = \sum_{v_i} w_{Fi}^p (x_i - x_B)^q + \sum_{v_i} w_{Bi}^p (x_i - x_F)^q. \quad (3)$$

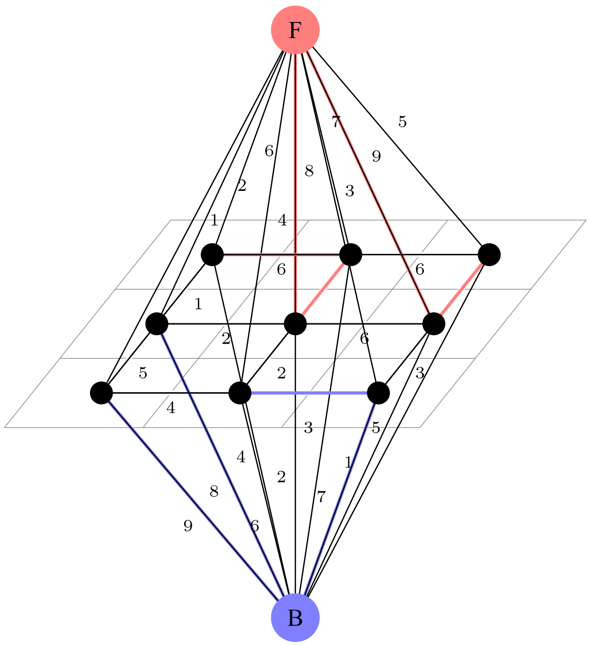
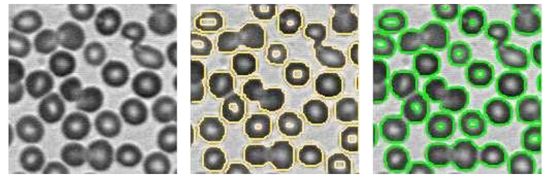


Figure 2: Example of unseeded segmentation of a 3×3 image computed a maximum spanning forest.

For the example of unseeded segmentation, the weights w_{Bi} between v_B and v_i can be fixed to reflect the agreement with the intensity at pixel v_i and a prior intensity model of the foreground and background intensities. A simple model that we use in this example is the difference between the pixel intensity and a learned mean intensity for both foreground and background. An example of such a weighted graph is given in Fig. 3. Given these unary terms, we may apply any of the algorithms of our framework on the resulting graph. An example of result is shown at Fig. 4.

6 Conclusion

We have proposed a general framework encompassing graph cuts, random walker, shortest path, and watersheds. In establishing the connection between the random walker and



(a) (b) (c)

Figure 3: Unseeded segmentation using unary terms. (a) Original image of blood cells, (b) Graph Cuts, (c) Power Watershed with q=2 performed without geodesic reconstruction. The result of segmentation with maximum spanning forest using Prim's or Kruskal's algorithm is very similar in practice because of the lack of plateaus in the gradient of the image.

optimal forest algorithms, we produced the power watershed algorithm (with $q = 2$) that have the property of providing a unique segmentation unlike most watersheds algorithms, is faster than graph cuts and random walker, can perform multi-seeds segmentation unlike graph cuts, and is more robust to seeds placement than shortest path forests.

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