Power Watersheds: A New Image Segmentation Framework Extending Graph Cuts, Random Walker and Optimal Spanning Forests

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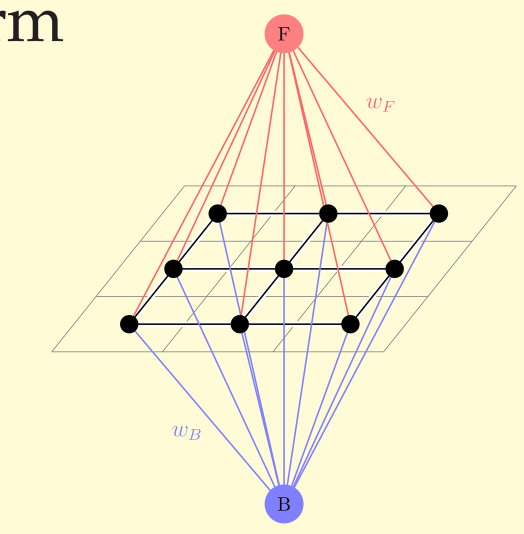
A Novel Unifying Energy Minimization Framework

We broaden the segmentation algorithm in [1] by separating the exponent on the weights (p) and the variables (q).

$$\min_x E_{p,q}(x) = \min_x \underbrace{\sum_{e_{ij}\in E} w_{ij}^p |x_i - x_j|^q}_{\text{smoothness term}} + \underbrace{\sum_{v_i} w_{Fi}^p |x_i|^q + \sum_{v_i} w_{Bi}^p |x_i - 1|^q}_{\text{data fidelity term}},$$

$$\text{subject to} \begin{cases} x(F) = 1, \\ x(B) = 0. \end{cases}$$

$$\text{Final segmentation given by } s_i = \begin{cases} 1 \text{ if } x_i \geq \frac{1}{2}, \\ 0 \text{ if } x_i < \frac{1}{2}. \end{cases}$$

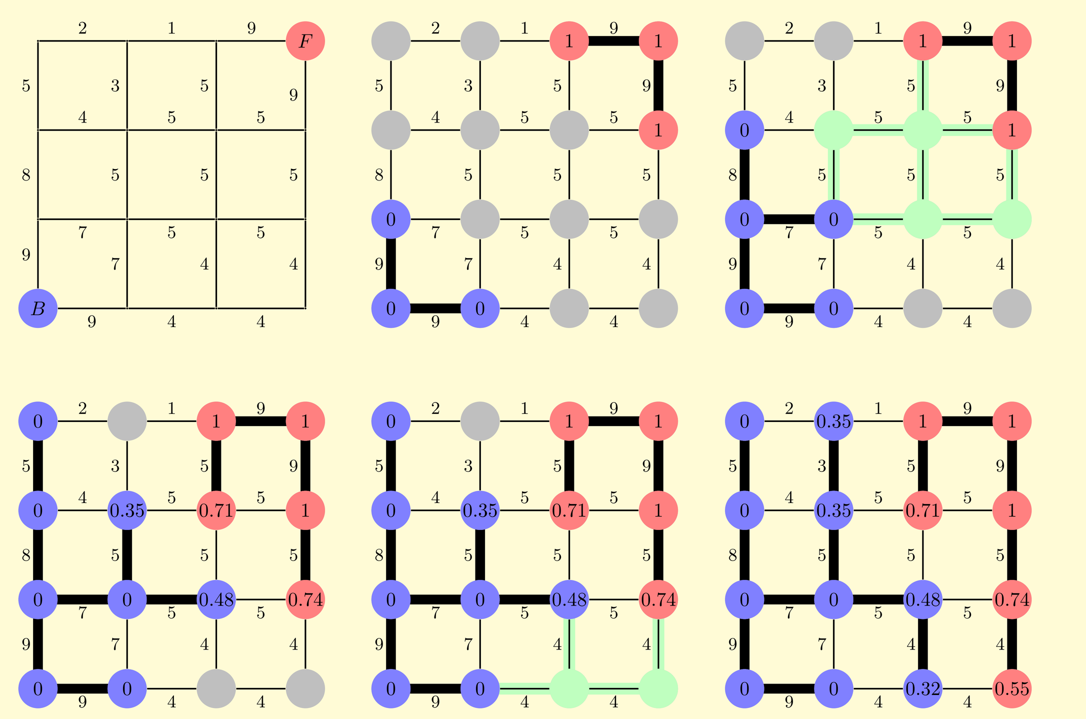


q \ p	0	finite	∞
1	Reduction to seeds	Graph cuts [1]	Maximum Spanning Forest (Watershed [3]) [2]
2	ℓ_2-norm Voronoi	Random walker [1]	Power watershed $q = 2$
∞	ℓ_1-norm Voronoi	ℓ_1-norm Voronoi	Shortest Path Forest [1]

Power Watershed Algorithm

Minimizing $E_{p,q}$, q finite, $p \to \infty$

1. Choose an edge of maximal weight e_{max}. Let S be the set of edges connected to e_{max} having the same weight as e_{max}.
2. If S already contains vertices that have different labels, minimize $E_{1,q}$ on S, otherwise merge the nodes of S into one node.
3. Repeat steps 1 and 2 until all vertices are labeled.



The case q finite, $p \to \infty$

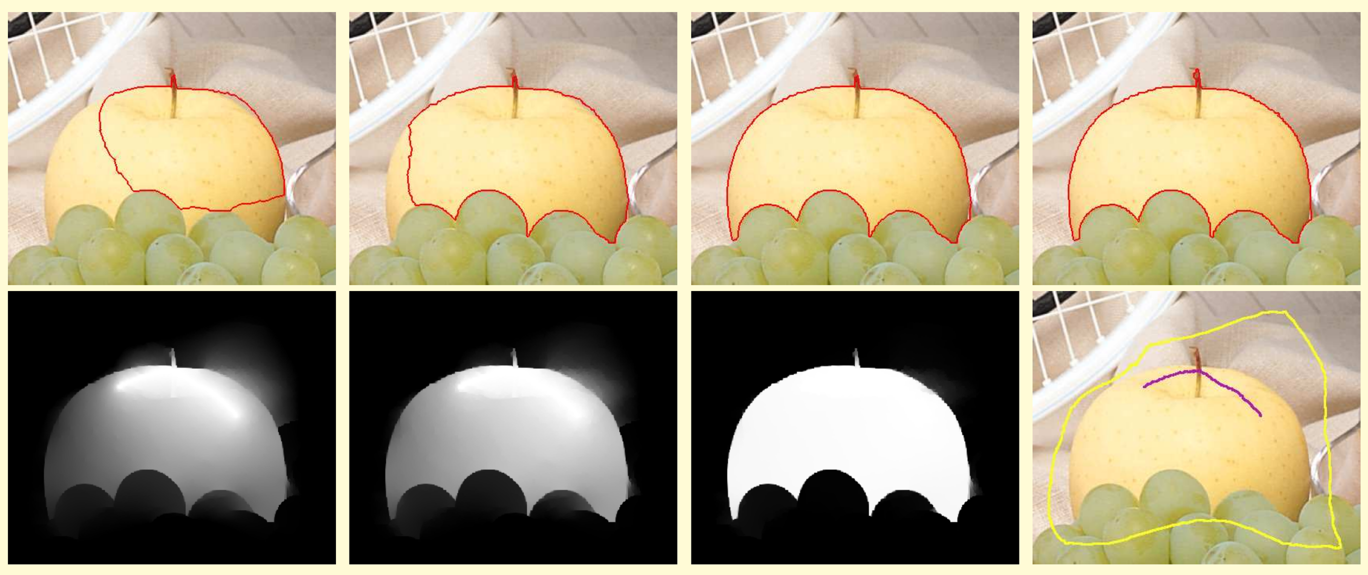
Theorem 1 *If the seeds are all the maxima of the weight function, the cut optimizing E_{pq} when $p \to \infty$ is a watershed cut.*

Property 1 *If $q > 1$, the solution x^* of $\min_x E_{p,q}(x)$ is unique.*

Theorem 2 (convergence) *If $q > 1$, the potential x^* converges toward $\bar{x}$ obtained by the Power watershed algorithm as $p \to \infty$.*

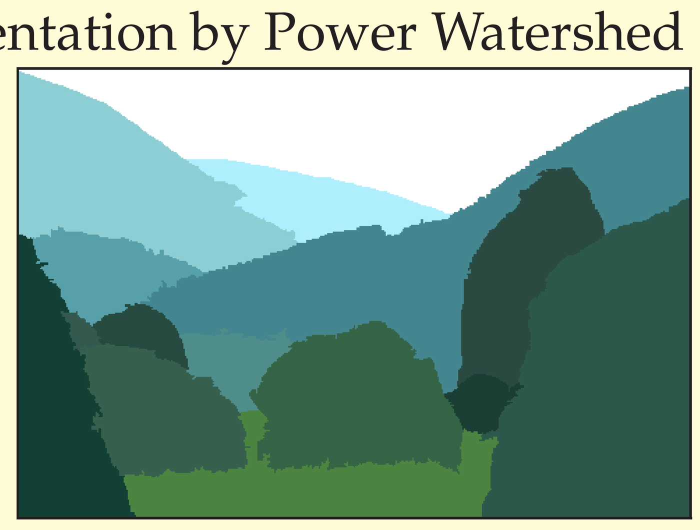
Illustration of convergence in the case $q = 2$, $p \to \infty$ [Thm. 2]

Progressive convergence to the power watershed as $p \to \infty$. Top row: Segmentations with $p = 1$, $p = 8$, $p = 25$ and power watershed. Bottom row: Corresponding potentials and seeds.



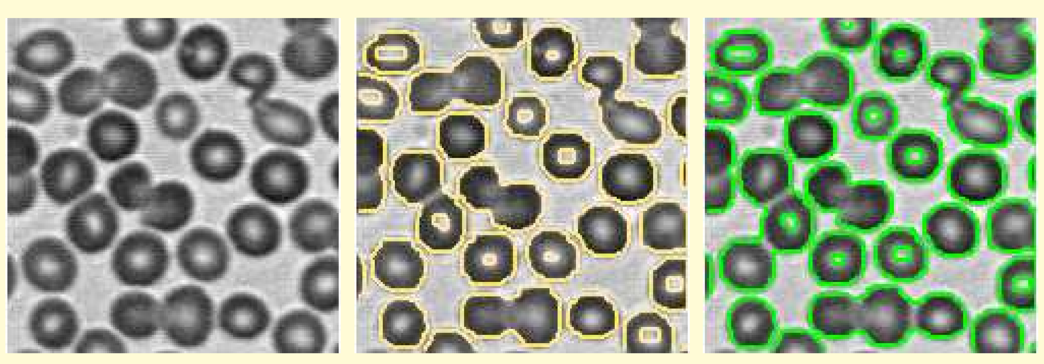
Exact Multiseed segmentation

Segmentation by Power Watershed ($q = 2$)



Example of use with unary terms

Segmentation of blood cells with Graph Cuts (yellow) and Power Watershed (green).



Comparative evaluation

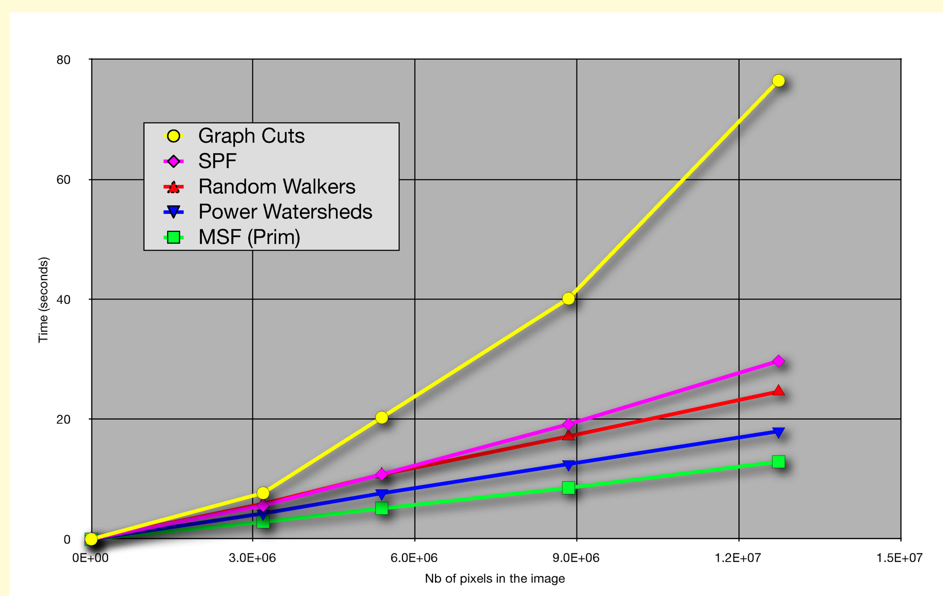
We compared the algorithms of our framework using a 2D vision database of images with ground truth and seeds from the "grabcut" project.

http://research.microsoft.com/en-us/um/cambridge/projects/visionimagevideoediting/segmentation/grabcut.htm

Graph cuts (yellow) Random walker (red) Shortest Path (magenta) Max Spanning Forest (Watershed) (green) Power watershed q=2 (blue)

	Dice coeff	Graph cuts	Random walker	Shortest paths	MaxSF	PW
Original set of seeds	mean	0.953	0.954	0.955	0.954	0.957
	stand. dev.	0.043	0.043	0.042	0.040	0.037
	med.	0.963	0.965	0.966	0.963	0.964
Thinned set of seeds	mean	0.925	0.921	0.918	0.922	0.924
	stand. dev.	0.061	0.064	0.062	0.062	0.064
	med.	0.933	0.934	0.932	0.935	0.937

Computation time 2D



Future work

Use power watersheds as a fast general-purpose optimizer in other vision problems.

References

[1] A. K. Sinop, L. Grady. A Seeded Image Segmentation Framework Unifying Graph Cuts and Random Walker Which Yields a New Algorithm. In *ICCV 2007*

[2] C. Allène, J-Y. Audibert, M. Couprie, J. Cousty, R. Keriven. Some links between min cuts, optimal spanning forests and watersheds. In *ISMM 2007*

[3] J. Cousty, G. Bertrand, L. Najman, M. Couprie. Watershed cuts: minimum spanning forests, and the drop of water principle. In *PAMI, 2009*