Fusion graphs, region merging and watersheds

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Problems

- Region merging methods consist of improving an initial segmentation by progressively merging pairs of neighboring regions.

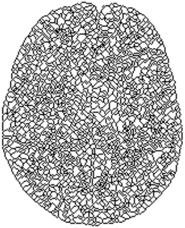
T.Pavlidis. *Structural Pattern Recognition*, chapters 4-5. Segmentation techniques, 1977.

Problems

In mathematical morphology region merging methods define hierarchies of watersheds:

- waterfall [BEUCHER94];
- geodesic saliency of watershed contours [NAJMAN96].

Problems



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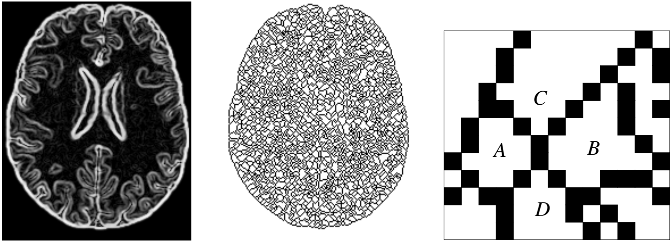

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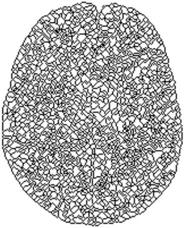

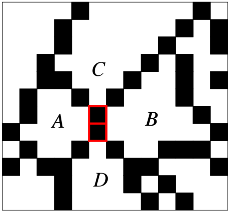

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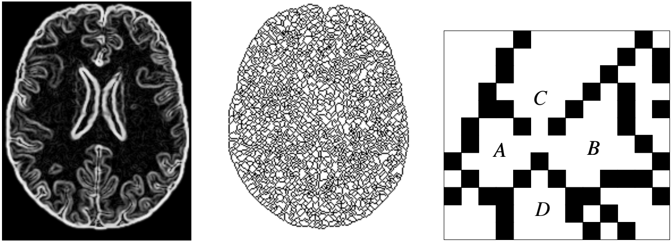

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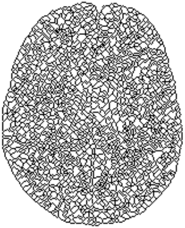
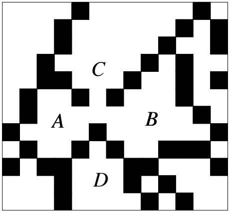




Problem

Is there some graphs in which any pair of neighboring regions can always be merged?

Fusion graphs, region merging and watersheds

Basic notions on graphs

Let $G = (E, \Gamma)$ be a graph, let $X \subseteq E$ and $Y \subseteq X$.

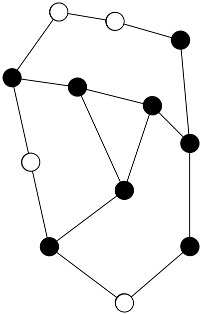
- We say that *X is connected* if $\forall x \in X, y \in X$, there exists a path from x to y in X.
- We say that Y is *a (connected) component of X* if Y is connected and if Y is maximal for this property.

Thin set

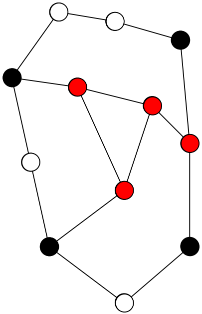
Let $X \subseteq E$.

- The *interior of X* is the set $\text{int}(X) = \{x \in X \mid \Gamma(x) \subseteq X\}$.
- we say that *X is thin* if $\text{int}(X) = \emptyset$.

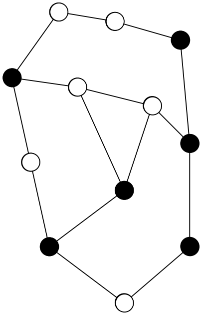
Interior, example



Interior, example



Thin set, example



- A set X separate its complementary set ($\overline{X}$) into connected components called regions for X.

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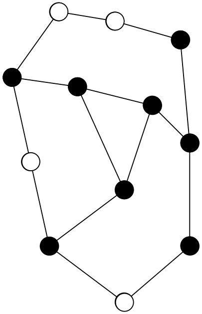

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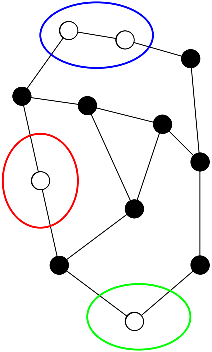

Watershed, introduction

- A watershed is a set of vertices which cannot be reduced without changing the number of regions.

Watershed: a model of frontier

Let $X \subseteq E$ and $p \in X$

- We say that p is *W-simple for X* if p is adjacent to exactly one region for X.

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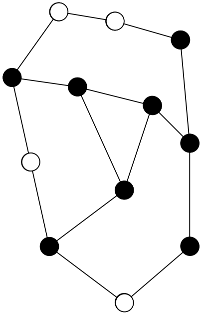

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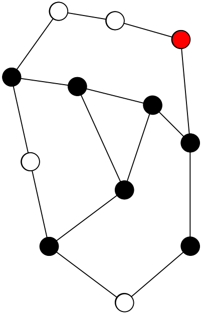

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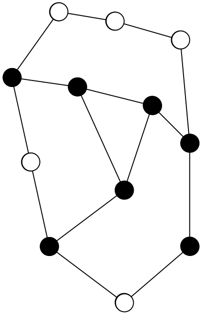
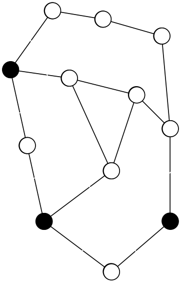


Watershed: a model of frontier

- The set X is a *watershed* if there is no W-simple point for X.

Watershed: example

Example of a thin watershed:



Watershed: example

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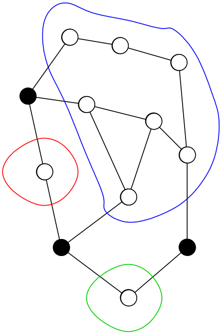
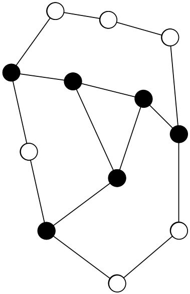
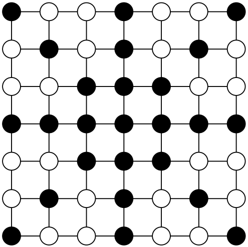


Watershed: example

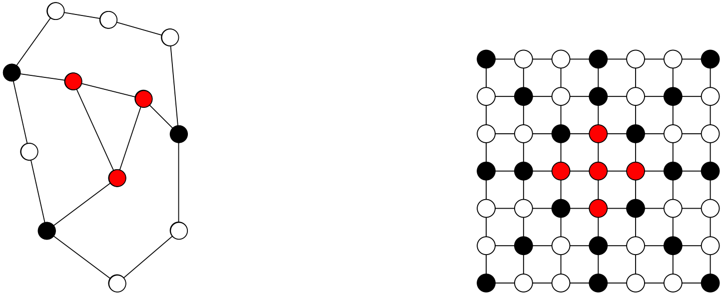
Non thin watershed:





Watershed: example

Non thin watershed:



- In applications, a thick watershed is awkward.

Problem

Is there some graphs in which any watershed is necessarily thin?

Region merging

Let A and B be two regions for X.

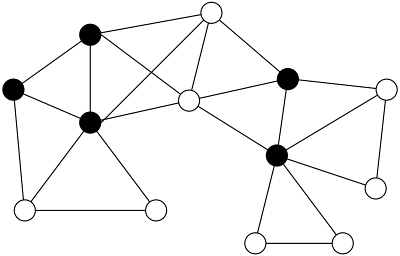
Definition

We say that A and B can be merged *if there exists* $S \subseteq X$ *such that :*

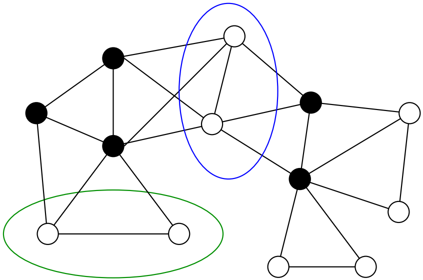
- A *and* B *are the only two regions for* X *adjacent to* S*; and*
- S *is connected.*

We also say that A and B can be merged through S.

Region merging, example



Region merging, example



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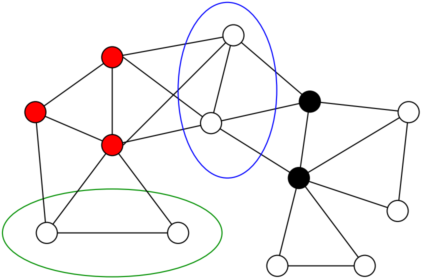
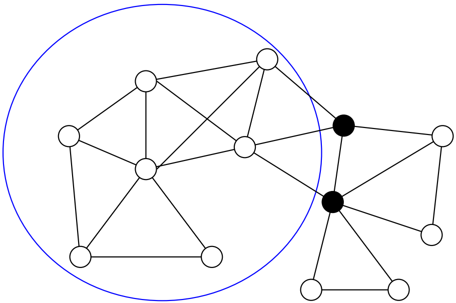
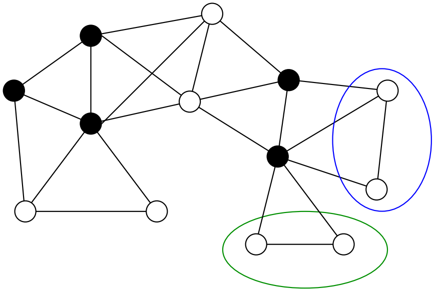


Region merging, example



Regions which cannot be merged, example



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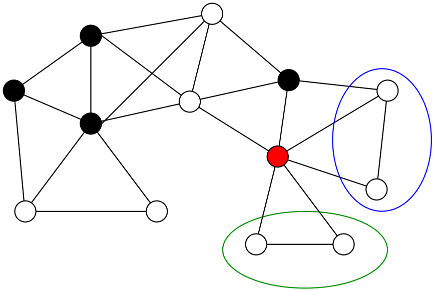
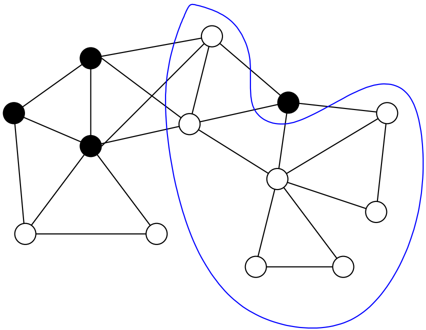


Regions which cannot be merged, example



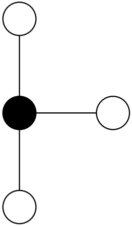
Region merging

We say that a region A *can be merged* if there exists a region B such that A and B can be merged.

General idea

- Characterization of the difficulties for defining region merging procedures.
- Definition of 4 classes of graphs in which these difficulties are avoided.

Unspecified graph: example



Difficulty

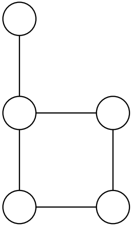
No region can be merged.

Weak fusion graph

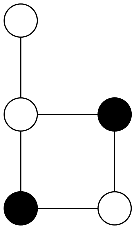
Definition

We say that a graph is a weak fusion graph if for any subset of the vertices, there exists a region which can be merged.

Weak fusion graph: illustration



Weak fusion graph: illustration



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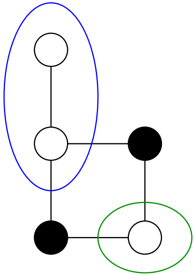

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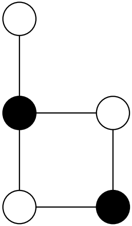

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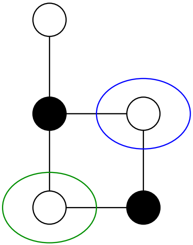

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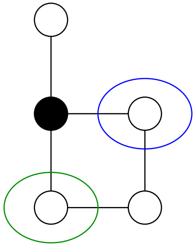

Weak fusion graph: limitation

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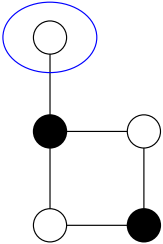

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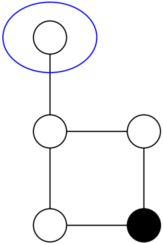

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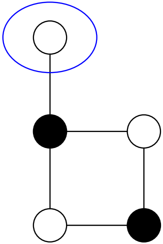


Difficulty

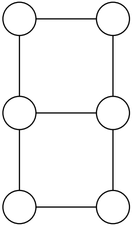
Some regions cannot be merged.

Fusion graph

Definition

We say that a graph is a fusion graph if for any subset of vertices, any region can be merged.

Fusion graph: illustration



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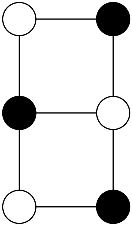

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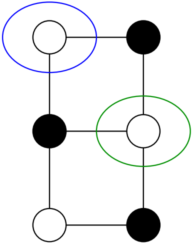

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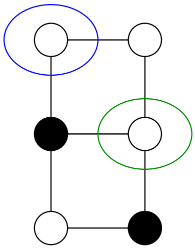

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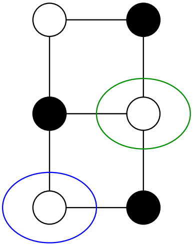

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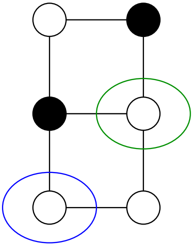

Fusion graph: limitation

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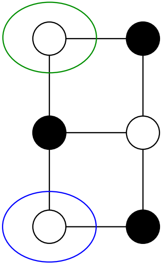

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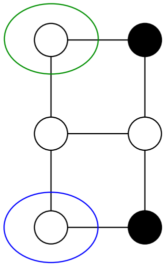

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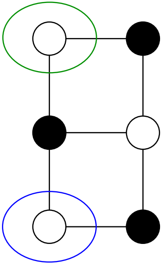


Difficulty

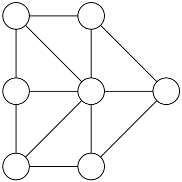
Some pairs of neighboring regions cannot be merged.

Strong fusion graph

Definition

We say that a graph is a strong fusion graph if for any subset of the vertices, any two regions which are neighbor can be merged.

Strong fusion graph: illustration



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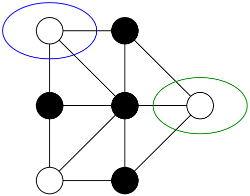

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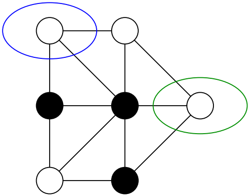
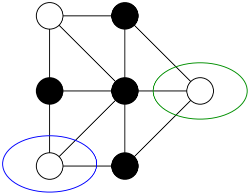


Strong fusion graph: illustration



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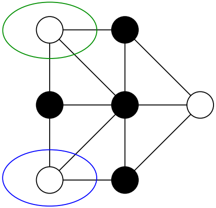
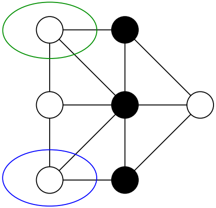


Strong fusion graph: illustration



Strong fusion graph: limitation

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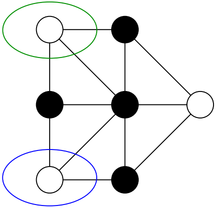

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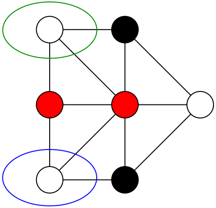

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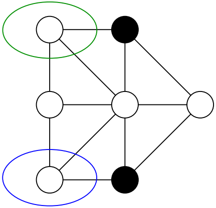

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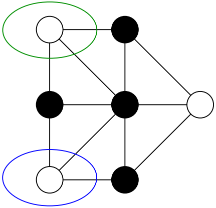


Difficulty

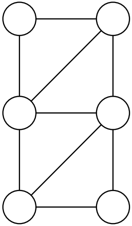
Some pairs of neighboring regions cannot be merged through their common neighborhood.

Perfect fusion graph

Definition

We say that a graph is a perfect fusion graph *if for any subset of the vertices, any two regions which are neighbor can be merged through their common neighborhood.*

Perfect fusion graph, example



Relations between the fusion graphs

We denote by $\mathcal{G}$ (resp. $\mathcal{G}_W$, $\mathcal{G}_F$, $\mathcal{G}_S$, $\mathcal{G}_P$) the set of all graphs (resp. weak fusion graphs, fusion graphs, strong fusion graphs, perfect fusion graphs).

Property

$\mathcal{G}_P \subset \mathcal{G}_S \subset \mathcal{G}_F \subset \mathcal{G}_W \subset \mathcal{G}$.

Characterization of fusion graph

Fusion graphs / thin watershed

Theorem

A graph G is a fusion graph if and only if any non-trivial watershed in G is thin.

Seven characterizations of perfect fusion graphs

Seven characterizations of perfect fusion graphs

Theorem

G is a perfect fusion graph if and only if any connected subgraph of G is a fusion graph.

Seven characterizations of perfect fusion graphs

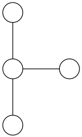
Theorem

G is a perfect fusion graph if and only if for any watershed in G, any point of the watershed is adjacent to exactly two regions.

Seven characterizations of perfect fusion graphs

Theorem

G is a perfect fusion graph if and only if the graph $G^{\blacktriangle}$ is not a subgraph of G.



- Based on the definitions, to check whether a graph G belongs to one of the classes of fusion graph, we need to analyze all subsets of the vertices of G.
- Exponential complexity.

Question

Is there some simple and polynomial characterizations of these classes of graphs?

Local characterizations

Theorem

- *There is no local characterization of weak fusion graph.*

Local characterizations

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- *There is no local characterization of fusion graph.*

Local characterizations

Theorem

- *There is no local characterization of weak fusion graph.*
- *There is no local characterization of fusion graph.*
- *There exists a local characterization of strong fusion graphs that allows to test in polynomial time if a given graph is a strong fusion graph.*

Local characterizations

Theorem

- *There is no local characterization of weak fusion graph.*
- *There is no local characterization of fusion graph.*
- *There exists a local characterization of strong fusion graphs that allows to test in polynomial time if a given graph is a strong fusion graph.*
- *There exists a local characterization of perfect fusion graphs that allows to test in polynomial time if a given graph is a perfect fusion graph.*

Question

In image processing, which adjacency relation can be used?
2D: Γ_4, Γ_8 *?*
3D: Γ_6, Γ_{26} *?*

Usual grids : classification

Theorem

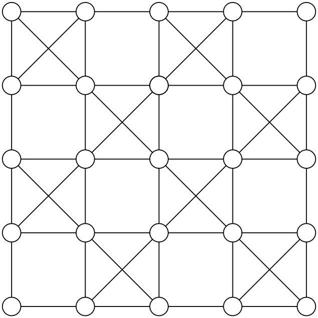
	weak fusion	fusion	strong fusion	perfect fusion
2D:Γ_4	no	no	no	no
3D:Γ_6	no	no	no	no
2D:Γ_8	yes	yes	no	no
3D:Γ_{26}	?	no	no	no

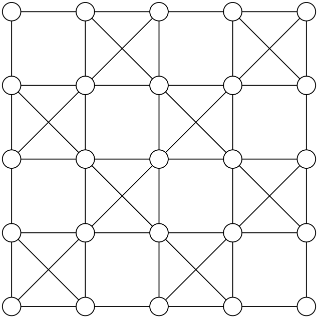
Perfect fusion grid

Question

Is there any adjacency relation on $\mathbb{Z}^2$, $\mathbb{Z}^3$,..., $\mathbb{Z}^n$ that induce perfect fusion graphs?

Perfect fusion grid, construction





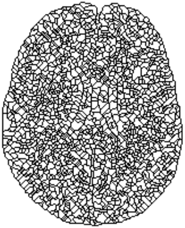
This construction can be generalized in dimension *n*, for any integer *n*.

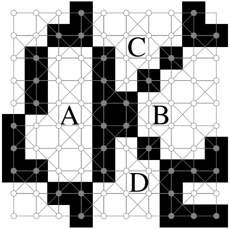
Perfect fusion grid: uniqueness

Theorem

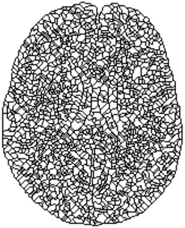
The perfect fusion grid on $\mathbb{Z}^n$ is the unique perfect fusion graph (up to a translation) which is between the usual adjacency relations (Γ_4^n, Γ_8^n).

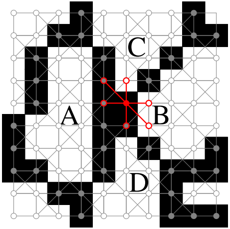
To conclude by an example ...



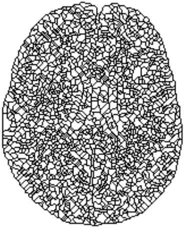


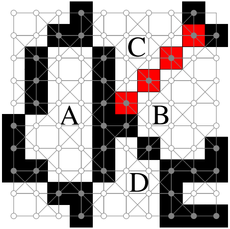
To conclude by an example ...



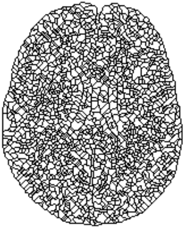


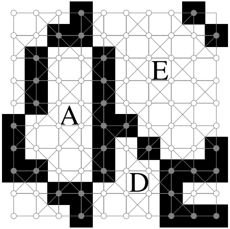
To conclude by an example ...





To conclude by an example ...





Conclusions

- **Introduction of a theoretical framework** that allows to understand and analyze, **region merging methods**
- **Classification of the usual adjacency relations** with respect to their merging properties
- **Definition of a new adjacency relation**, on $\mathbb{Z}^n$, adapted for region merging procedure.

Perspectives

- **Grayscale watersheds on perfect fusion graphs** (IWCIA06):
  - Thinness of grayscale topological watershed
  - Linear-time immersion-like algorithm
- **Region merging schemes**:
  - Morphological criterion, saliency, and watershed hierarchies
  - Links between minimum spanning trees and watersheds
- **Watershed on edges**:
  - Drop of water principle
  - Optimality of watersheds
  - Algorithms

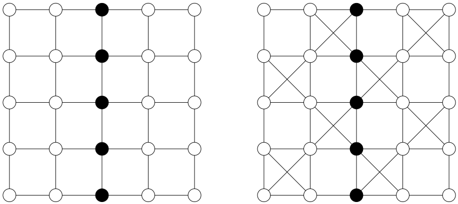
Référence

IGM 2005-04, J. Cousty, G. Bertrand, M. Couprie, L. Najman,*Fusion graphs: merging properties and watersheds*, Institut Gaspard Monge, 2005

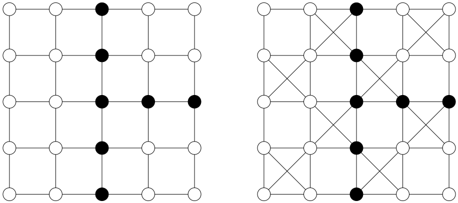
soumis à CVIU, special issue commemorating the career of Prof. Azriel Rosenfeld

http://igm.univ-mlv.fr/LabInfo/rapportsInternes/2005/04.pdf

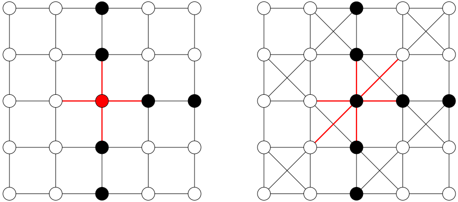
Region splitting



Region splitting



Region splitting



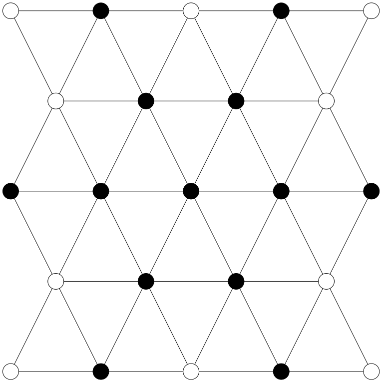
Region splitting

Property

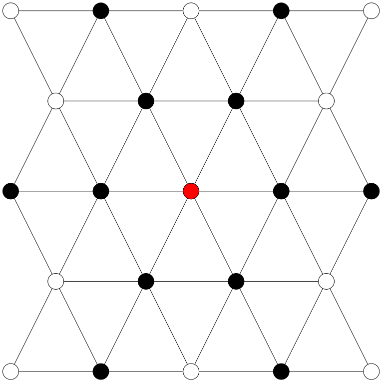
Let (E,Γ) be a perfect fusion graph. Let $X \subseteq E$, be a watershed A be a region for X. Let $Y \subseteq A$ be a watershed on $(A, \Gamma \cap [A \times A])$ then $X \cup Y$ is a watershed on (E,Γ).

The property is, in general, not true on non perfect fusion graph.

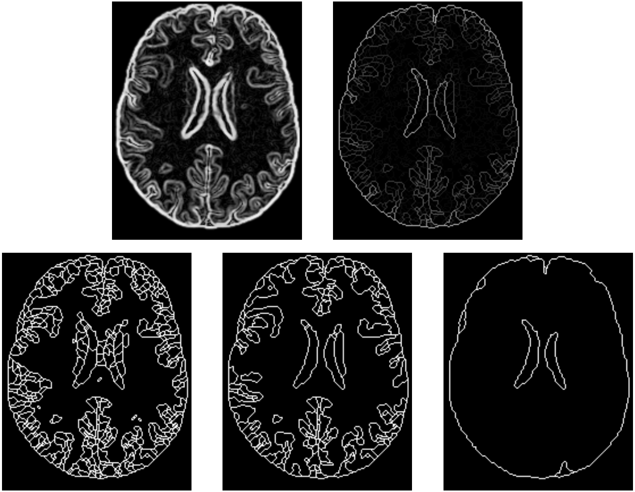
Hewagonal grid



Hewagonal grid



An example of Saliency



Perfect fusion grid in dimension 3