**Basic Concepts**

A hierarchy of partitions is a chain of increasing partitions of some finite set $E$. $H = \{\pi_i, 0 \leq i \leq n | i \leq k \leq n \Rightarrow \pi_i \leq \pi_k\}$.

And each node or partial partition in the hierarchy is given an energy $\omega : \pi(S) \rightarrow \mathbb{R}^+$.  

**Partition versus cuts:**

Number of possible cuts in the hierarchy increases more than exponentially with number of levels.

**Theorem:** When the energy is non-increasing and singular, then the temporary optimum at node $S$ is either $S$ itself or the union of the optimal cuts of the sons of $S$ [3].

This can be calculated by a bottom up dynamic program, that calculates the optimum at each partial partition $S$.

**Problems**

- Given a hierarchy $H$ and ground truth partition $G$ find the partition in $H$ closest to $H \rightarrow G$
- Closest from $G \rightarrow H$
- Compare any hierarchy $H$ with multiple ground truth partitions $G_i$ of image.
- Compare any two hierarchies $H_1, H_2$, with respect to a common partition $G$.

**Partitions in the hierarchy:**

![Input image and Ground truths](image)

**Local half Hausdorff measures: Ground truth energies**

**Hausdorff distance:** Smallest disc dilation of $X$ that contains $Y$ and of $X$ to contain $Y$

$$d_H(X, Y) = \max \{\sup_{x \in X} \inf_{y \in Y} d(x, y), \sup_{y \in Y} \inf_{x \in X} d(x, y)\}$$

**Problems:** Global measure, large variations when object are asymmetric w.r.t. each other.

**Different proximities:**

- $H \rightarrow G, G \rightarrow H, H \rightarrow G_i, H_i \rightarrow G$

**Optimal Cuts and composition laws**

**Composition law:** Supremum for $\omega_G$ and $\theta_G$, Infimum for $\omega_G + \theta_G$

**Different proximities:**

- $H \rightarrow G, G \rightarrow H, H \rightarrow G_i, H_i \rightarrow G$

**Optimal cuts:**

![Optimal cuts](image)

**Global Similarity measures to integrate the proximity between a sequence of partitions and a ground truth.**

$$P = \sum_{i=0}^{n} \frac{1}{N} \int_{\omega_i \in \omega(G)} (1 - g(x)) S_i(x) dx$$

$$R = \sum_{i=0}^{n} \frac{1}{N} \int_{\omega_i \in \omega(G) + \theta(G)} (1 - g(x)) S_i(x) dx$$

References:


[5] Theorem: When the energy is non-increasing and singular, then the temporary optimum at node $S$ is either $S$ itself or the union of the optimal cuts of the sons of $S$ [3].

This can be calculated by a bottom up dynamic program, that calculates the optimum at each partial partition $S$.

**Ground truth energies for hierarchies of segmentations**

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http://www.esiee.fr/ kiranr/HierarchyEvalGT.html