Morphological Filtering in Shape Spaces : Applications Using Tree-Based Image Representations

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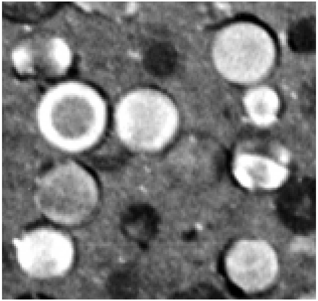
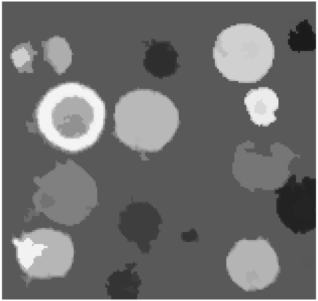
Motivation



Input image.



Result.

Question

How to obtain such a result?

Outline

Outline

Outline

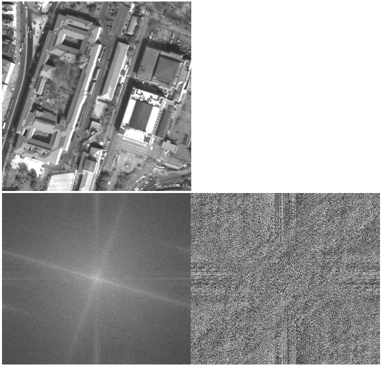
Outline

Outline

Image representations

Decomposition into primitive or fundamental elements that can be more easily interpreted:

- Functional decompositions;
- Multiresolution decompositions;
- Multi-scale representations;
- Threshold decompositions;
- Hierarchical representations.



Amplitude Phase

Image representations

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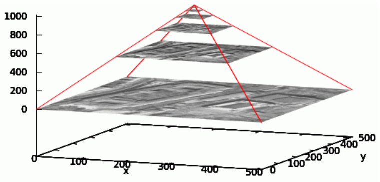

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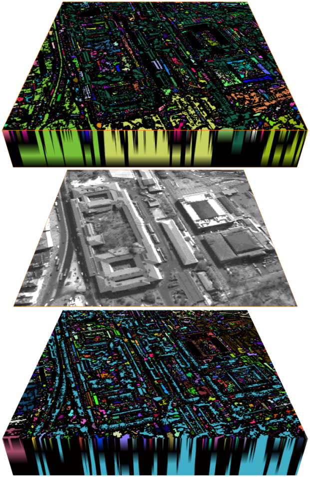

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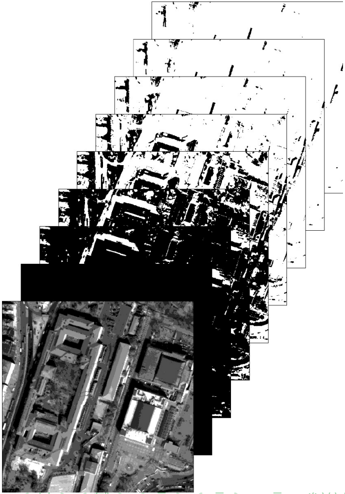

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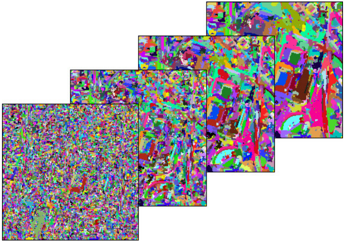


Image representations

Decomposition into primitive or fundamental elements that can be more easily interpreted:

- Functional decompositions;
- Multiresolution decompositions;
- Multi-scale representations;
- Threshold decompositions;
- Hierarchical representations.

Not mutually exclusive.
Properties inherited from those of underlying operations.
Choice driven by the application needs.

Connected operators

What's connected operators ?

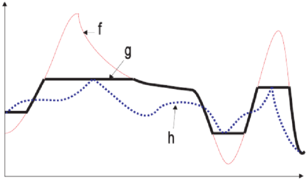
Filtering tools that merge flat zones.

Properties

- No new contours,
- Keep contours' position.

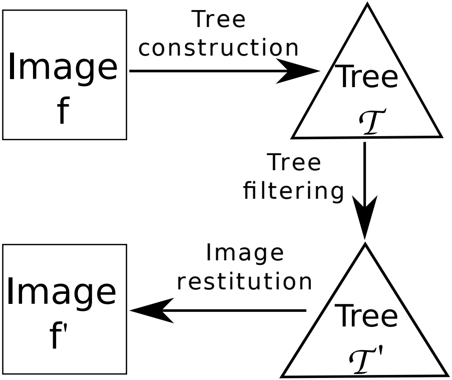
An example : Levelings

Lower-leveling: for x and y neighbors,
$g(x) > g(y) \Rightarrow g(y) \geq f(y)$.
Upper-leveling: for x and y neighbors,
$g(x) > g(y) \Rightarrow g(x) \leq f(x)$.
Leveling: Lower-leveling $\cap$ Upper-leveling.

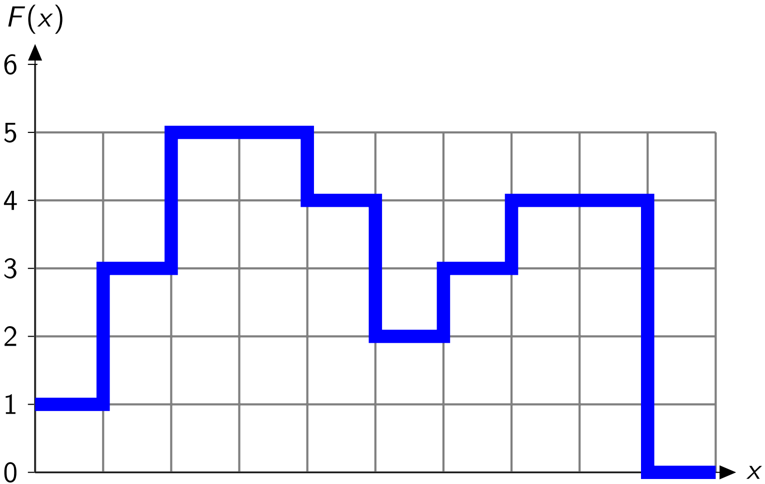


Leveling with marker.
f : input,
h : marker,
g : result.

One popular implementation [Salembier & Wilkinson, SPM, 2009]

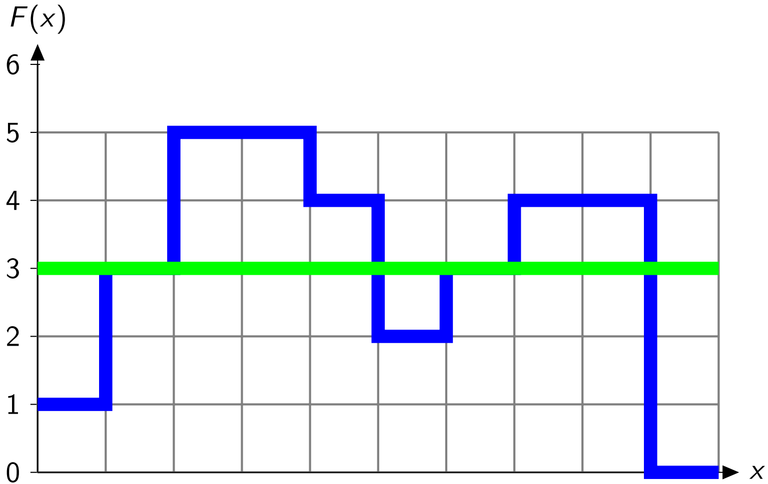


Level sets and components



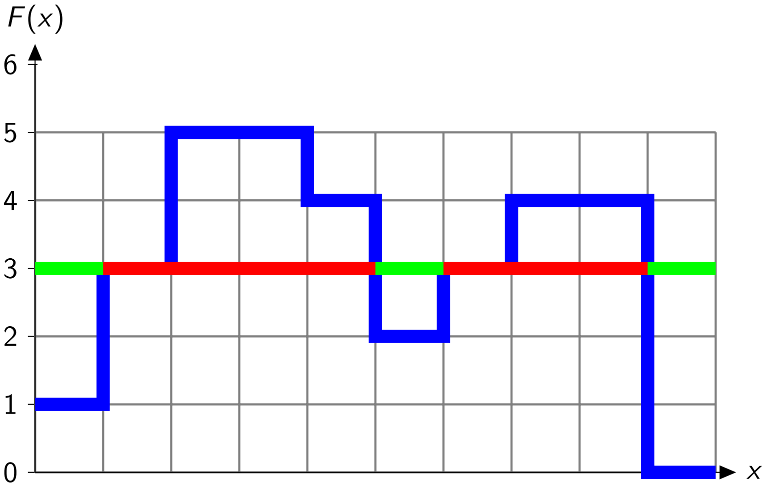
$$y = F(x).$$

Level sets and components



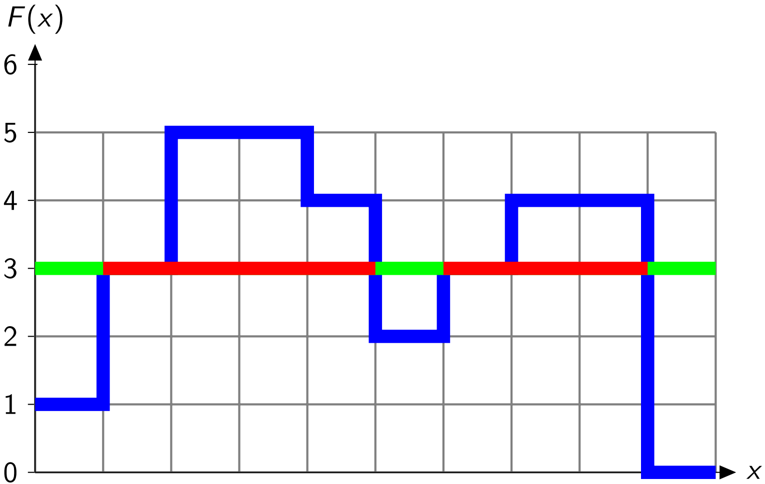
$F(x) \geq 3.$

Level sets and components



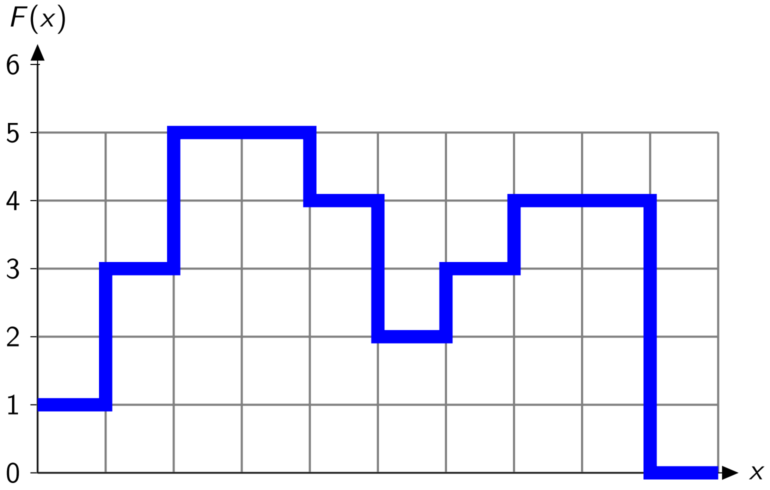
$$F_3 = \{x \mid F(x) \geq 3\}.$$

Level sets and components

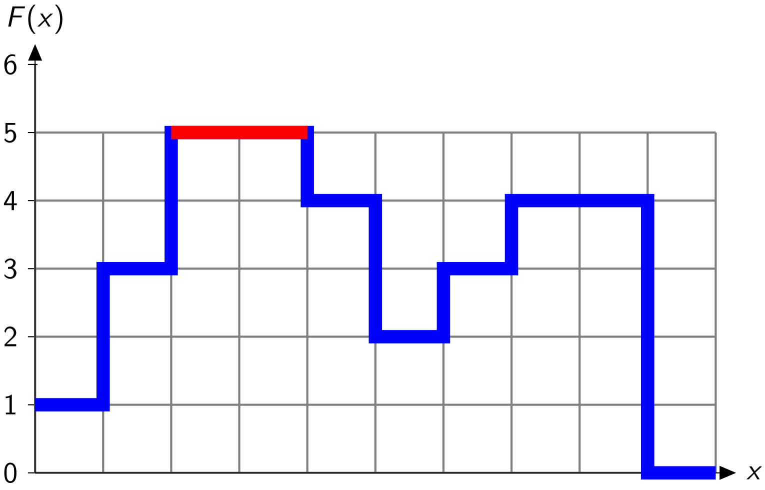


$$F_k = \{x \mid F(x) \geq k\}.$$

(Max) component tree

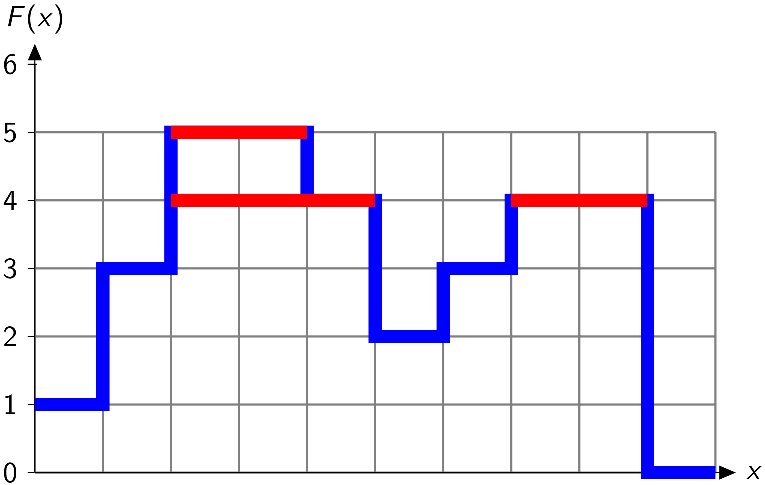


(Max) component tree



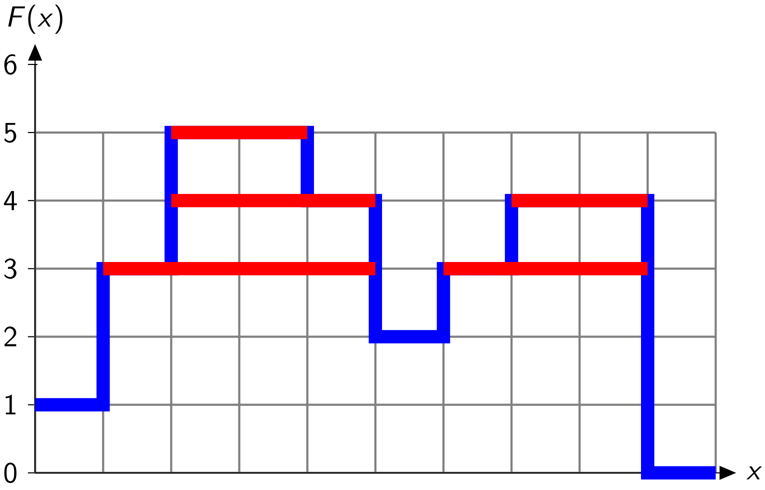
$$F_5 = \{x \mid F(x) \geq 5\}.$$

(Max) component tree



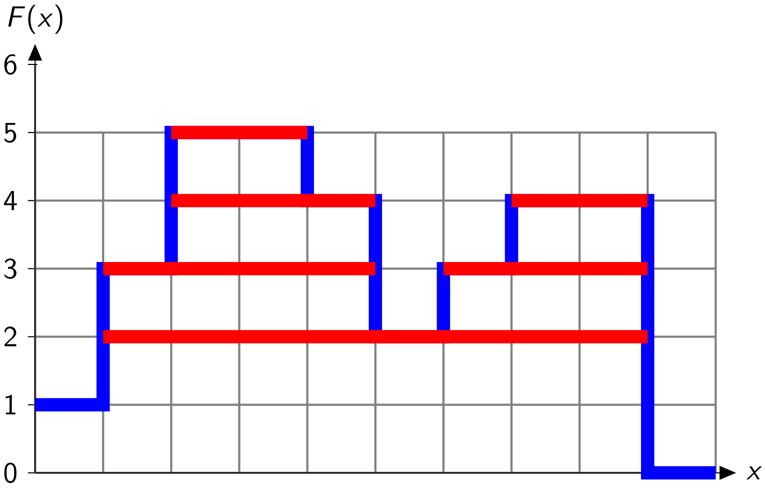
$$F_4 = \{x \mid F(x) \geq 4\}.$$

(Max) component tree



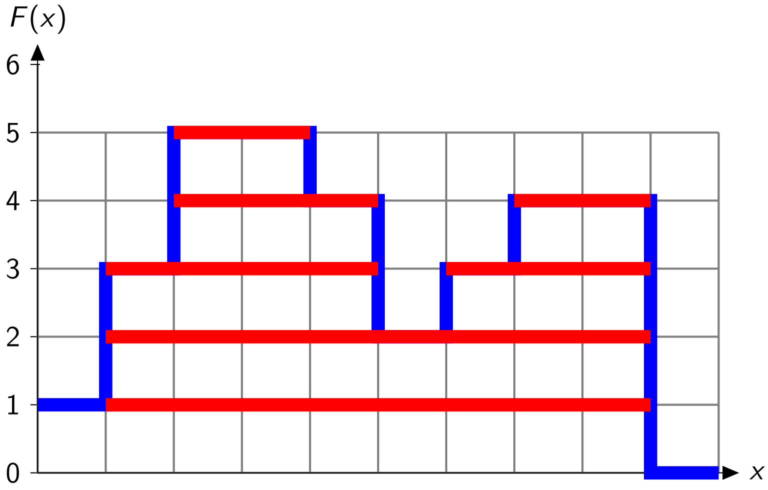
$$F_3 = \{x \mid F(x) \geq 3\}.$$

(Max) component tree



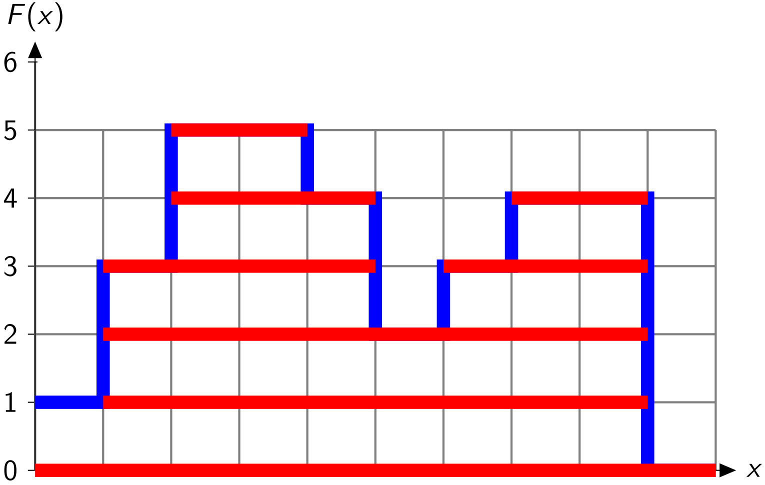
$F_2 = \{x \mid F(x) \geq 2\}$.

(Max) component tree



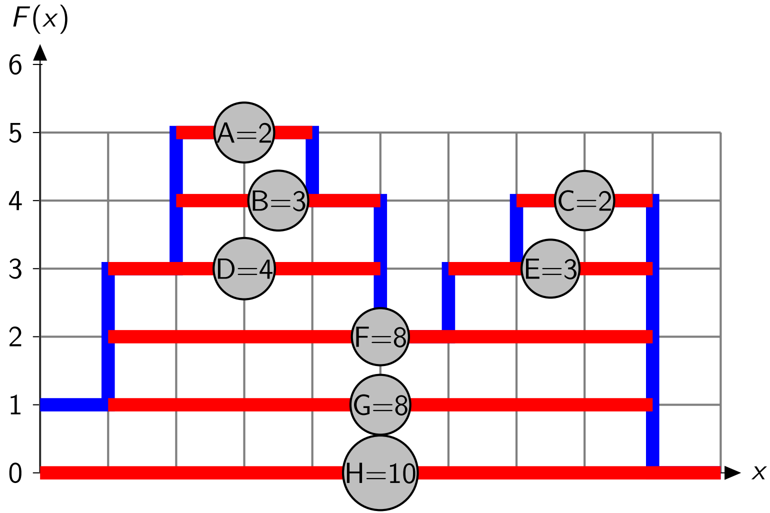
$F_1 = \{x \mid F(x) \geq 1\}$.

(Max) component tree

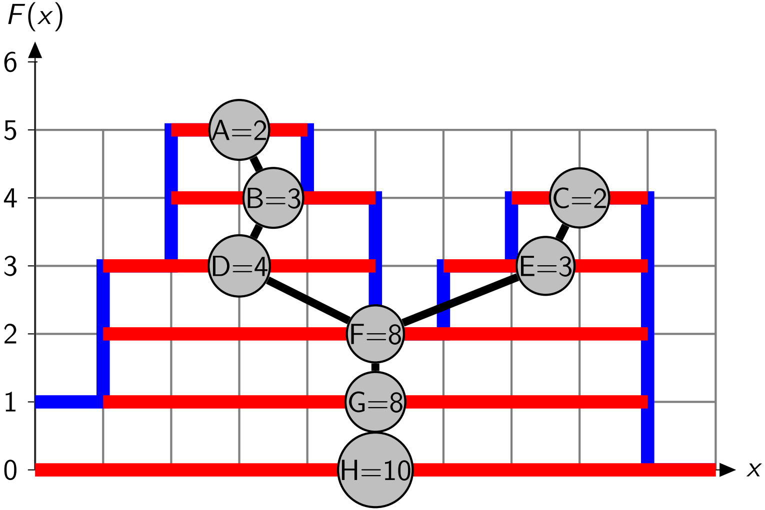


$$F_0 = \{x \mid F(x) \geq 0\}.$$

(Max) component tree

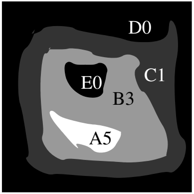


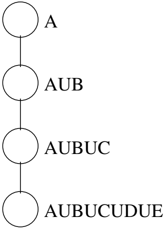
(Max) component tree



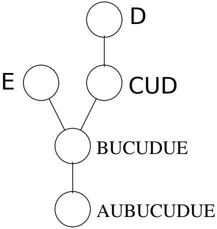
Components + inclusion relationship = component tree.

Min-tree, max-tree and tree of shapes [Monasse, ITIP, 2000]

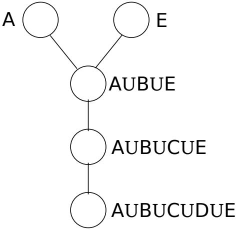




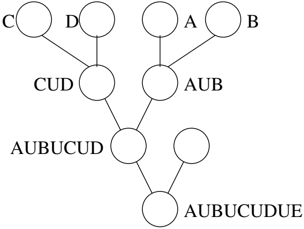
Max–tree



Min–tree

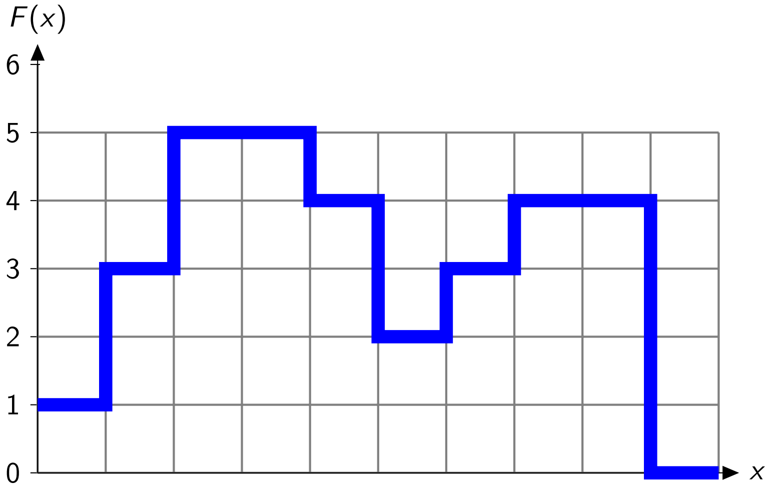


Tree of shapes

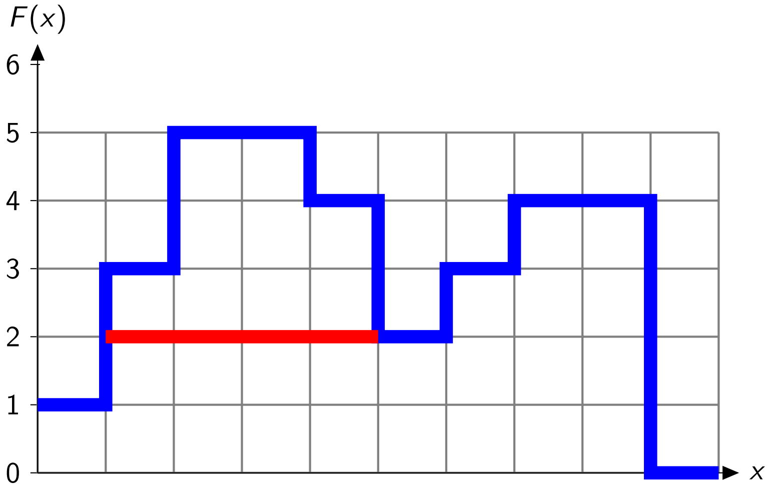


Binary Partition Tree

Attributes

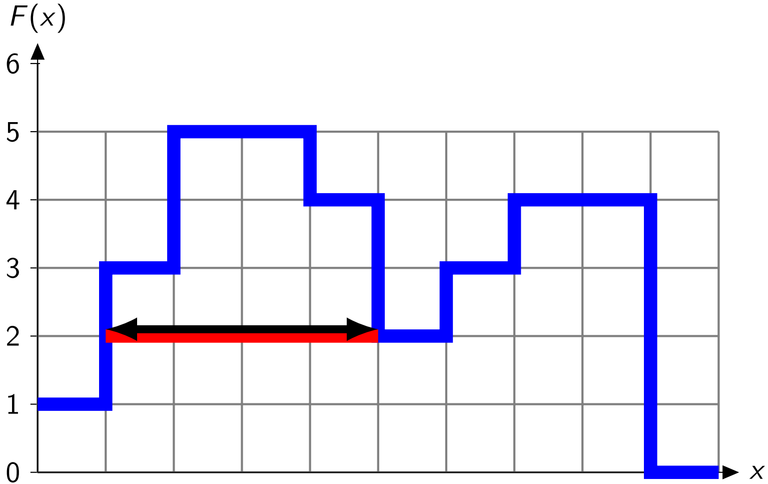


Attributes



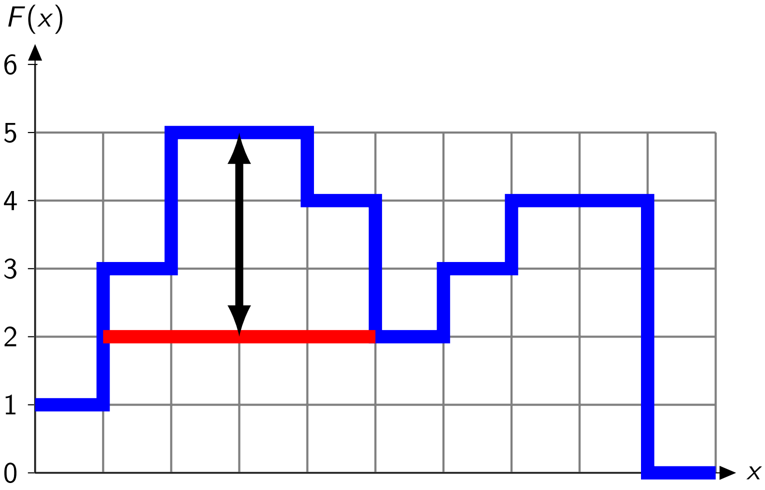
A connected component.

Attributes



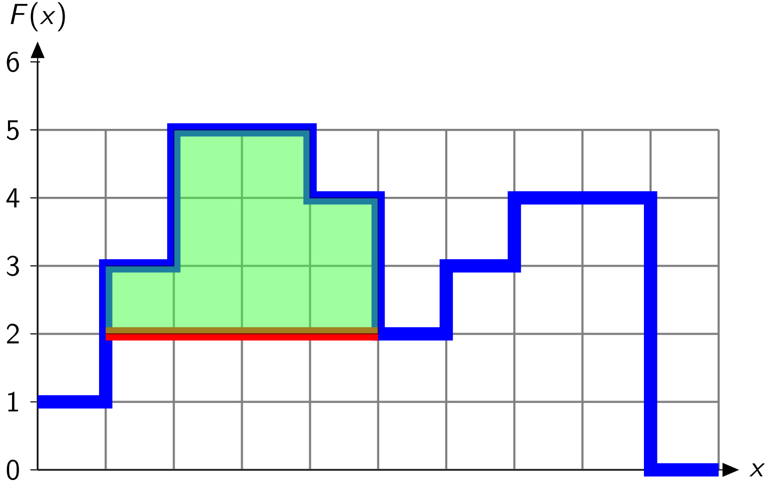
Area.

Attributes



Height.

Attributes



Volume.

Attributes

Increasing attributes

Increasing attributes : $A \subseteq B \Rightarrow \mathcal{A}(A) \leq \mathcal{A}(B)$.
Examples : Area, height, volume.

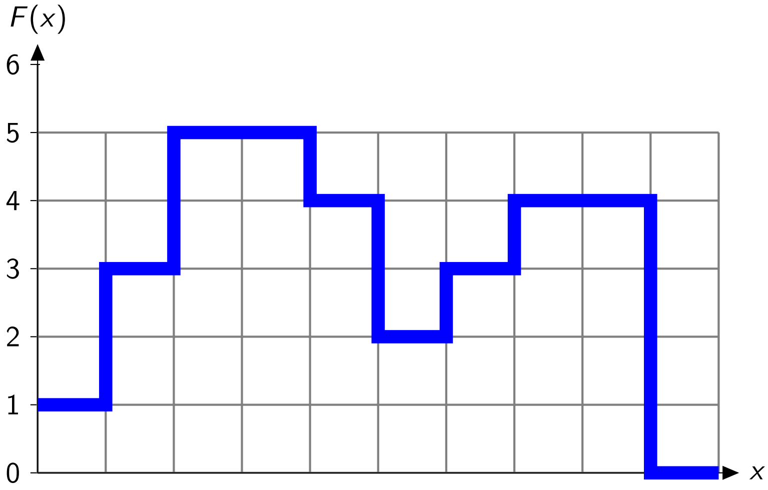
Non-increasing attributes

Shape attributes.

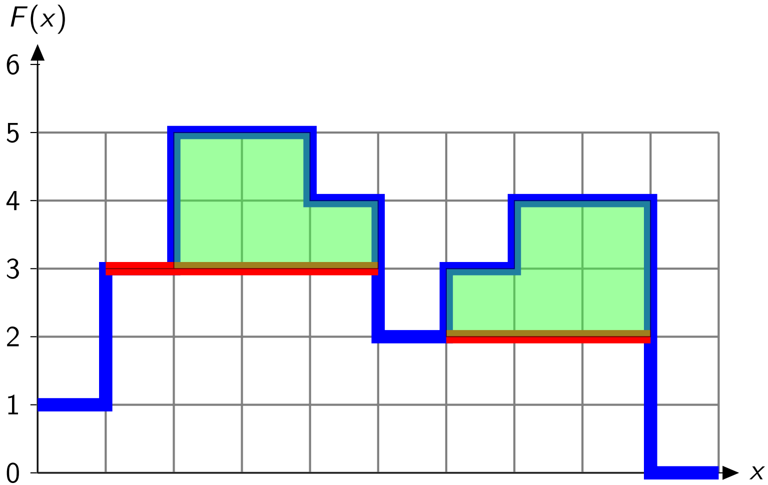
- I/A^2 minimum for a round object,
- Circularity : $area/(\pi \times l_{max}^2)$,
- Elongation : L_{max}/L_{min}.

L_{min} and L_{max} : Length of the two main axes of the best fitting ellipse.

Filtering with increasing attributes

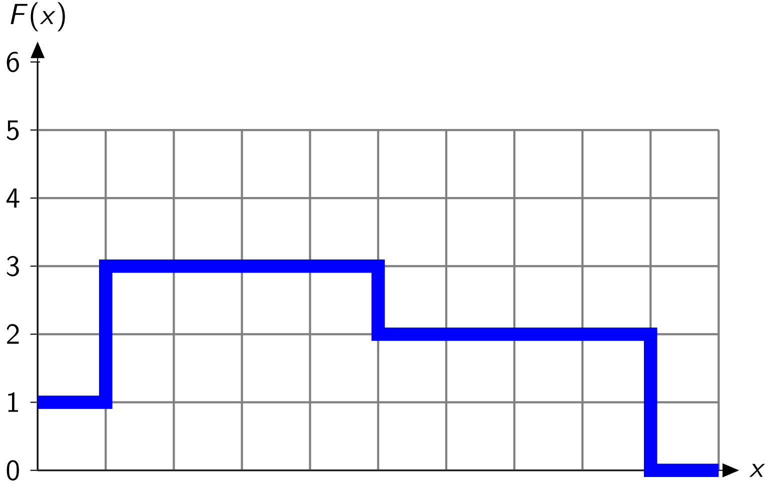


Filtering with increasing attributes



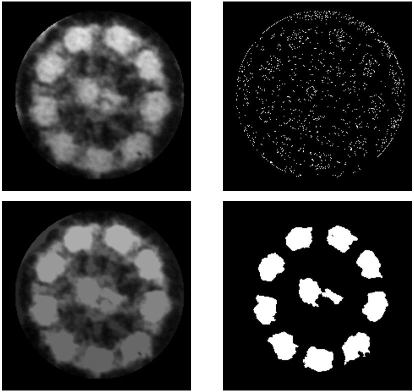
Volume ≤ 5.

Filtering with increasing attributes



Filtered function.

Application: Filtering with increasing attribute



Question

- *Increasing criterion (here, volume),*
- *How to process non-increasing criteria?*

Filtering with increasing attributes

Pruning the trees

$\mathcal{A} \uparrow$, Pruning the leaves = Attribute thresholding.

Non-increasing attributes

How to process the filtering?

Filtering with non-increasing attributes [Salembier & Wilkinson, SPM, 2009]

Pruning strategies

- *Min*,
- *Max*,
- *Viterbi*.

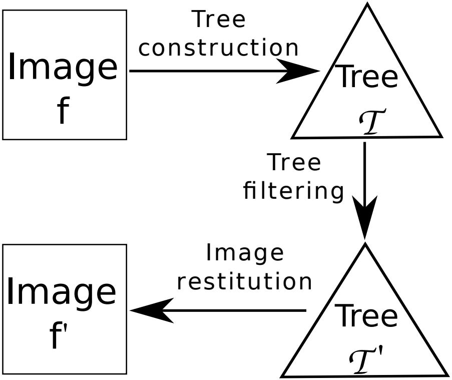
Remove the sub-tree rooted in the node.

Attribute thresholding strategies

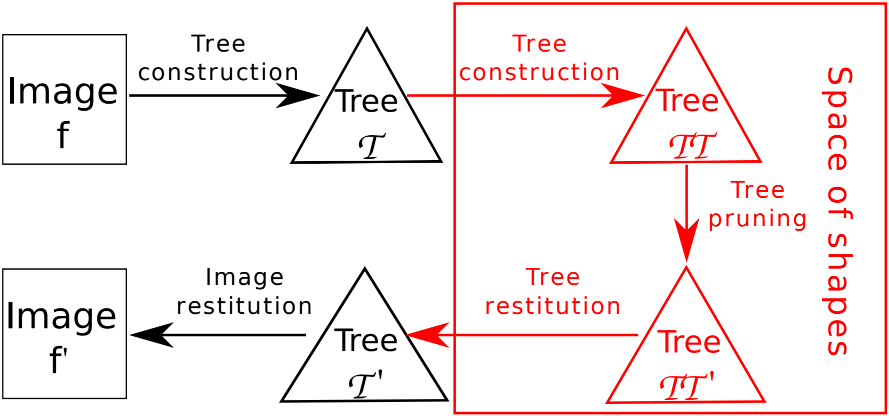
- *Direct*,
- *Subtractive*.

Remove the nodes under the threshold.

Our proposed framework

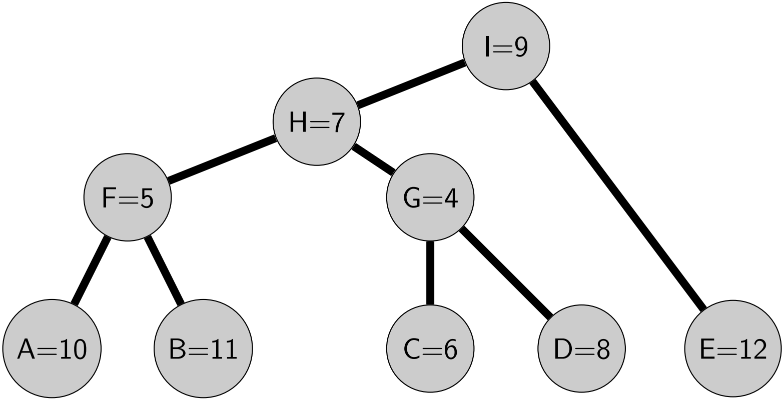


Our proposed framework [Xu & Géraud & Najman, ICPR, 2012]



Outline

Construction of second tree representation



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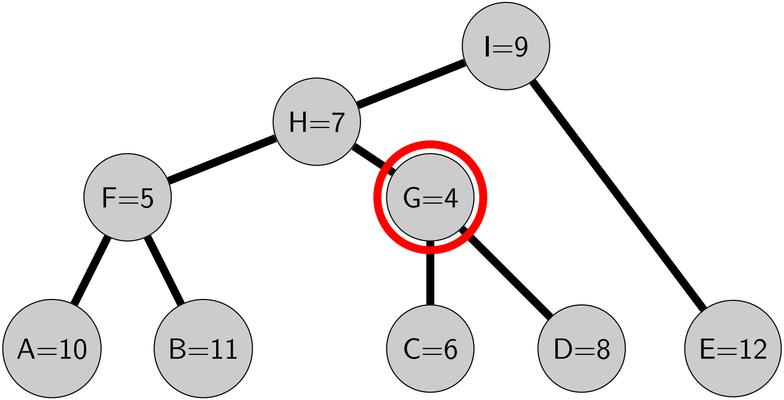
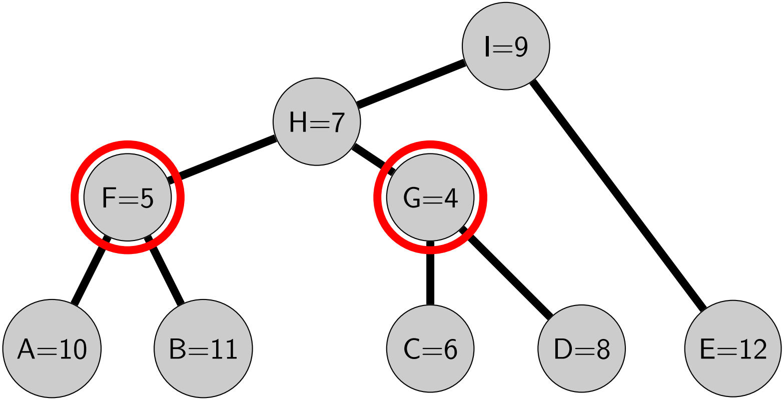


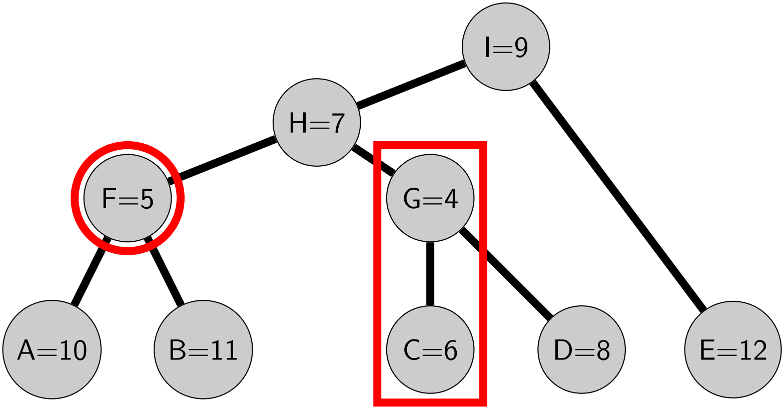
Level $\{x|A(x) \leq 4\}$.

Construction of second tree representation



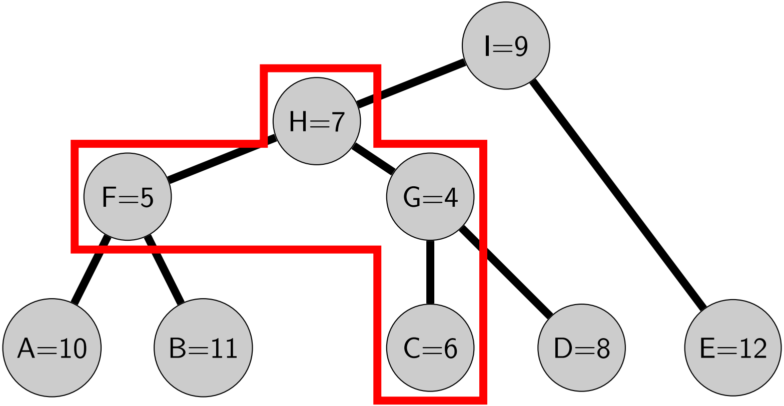
Level $\{x|A(x) \leq 5\}$.

Construction of second tree representation



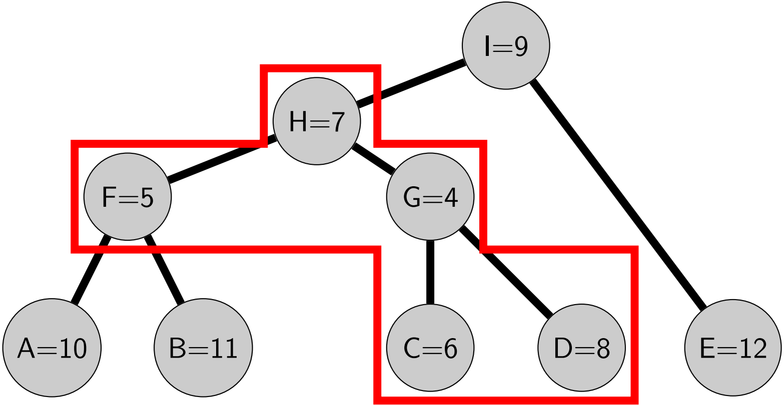
Level $\{x | A(x) \leq 6\}$.

Construction of second tree representation



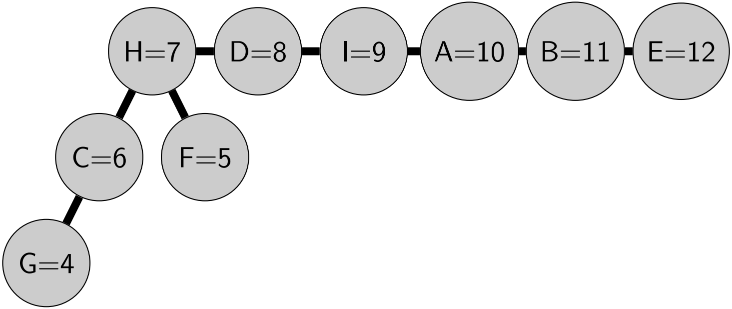
Level $\{x | A(x) \leq 7\}$.

Construction of second tree representation



Level $\{x|A(x) \leq 8\}$.

Min-tree of a tree-based image representation



Important idea

Computing a Min-Tree on a node-weighted graph instead of a matrix image.
Easy thanks to Olena [Levillain & Géraud & Najman, ICIP, 2010], *the generic image processing platform* `http://olena.lrde.epita.fr`.

Encompassing classical attribute filtering strategies

Increasing attribute $\mathcal{A}$

$\mathcal{T}\mathcal{T} = \mathcal{T}$.
No need to check if the attribute is increasing or not.

Attribute thresholding for non-increasing $\mathcal{A}$

$\mathcal{A}\mathcal{A} = \mathcal{A}$,
$\mathcal{A}\mathcal{A}$ is the current level of $\mathcal{T}\mathcal{T}$.
Pruning $\mathcal{T}\mathcal{T}$ = Attribute thresholding.

Shape-based lower/upper-levelings

Shape-based lower-levelings

$\mathcal{T}$: Max-tree,

$\forall x \in E, \psi_s(f)(x) \leq f(x)$ always holds $\Rightarrow \psi_s(f)$ is a lower-leveling of f.

$\Rightarrow$ **Shape-based lower-levelings.**

Shape-based upper-levelings

$\mathcal{T}$: Min-tree,

$\forall x \in E, \psi_s(f)(x) \geq f(x)$ always holds $\Rightarrow \psi_s(f)$ is a upper-leveling of f.

$\Rightarrow$ **Shape-based upper-levelings.**

Morphological shapings

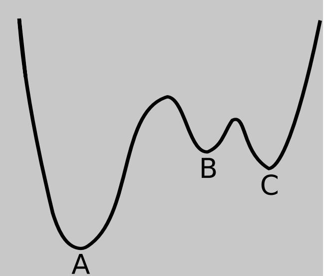
Morphological shapings

$\mathcal{T}$: Tree of shapes,

The order between $\psi_s(f)$ and f no more guaranteed, not levelings, but it is self-dual.

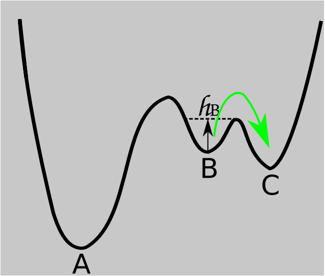
$\Rightarrow$ **Self-dual morphological shapings**.

Extinction-based filtering strategy



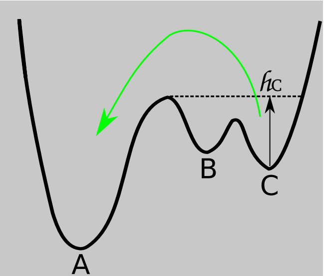
Given a strict order for the set of minima : $A \prec C \prec B$.

Extinction-based filtering strategy



B merges with *C*.

Extinction-based filtering strategy



C merges with A.

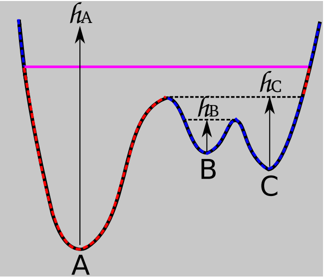
Extinction-based filtering strategy

Strategy

Preserve the **blobs of minima** whose extinction value > a given value.

Advantage

Only the connected components being meaningful enough compared with their context are preserved.



Extinction value of three minima.

Application to object segmentation

Context-based estimator for object detection

[Xu & Géraud & Najman, ICIP, 2012]

$$E(u, \partial\tau) = E_{int}(u, \partial\tau) + E_{ext}(u, \partial\tau) + E_{con}(u, \partial\tau).$$

$$V(u, \mathcal{R}) \;=\; \sum_{p \in \mathcal{R}} \left(u(p) - \overline{u}(\mathcal{R})\right)^2,$$

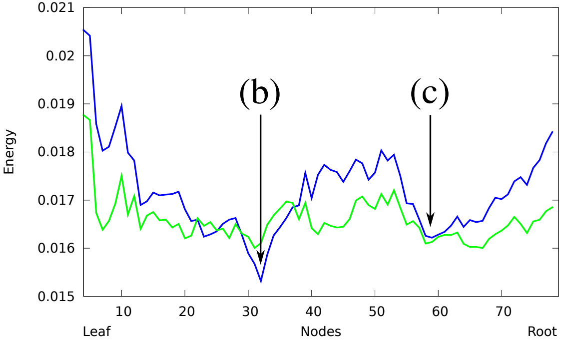
$$E_{ext}(u, \partial\tau) \;=\; \frac{V\big(u,\, \mathcal{R}^{\varepsilon}_{in}(\partial\tau)\big) \;+\; V\big(u,\, \mathcal{R}^{\varepsilon}_{out}(\partial\tau)\big)}{V\big(u,\, \mathcal{R}^{\varepsilon}_{in}(\partial\tau) \cup \mathcal{R}^{\varepsilon}_{out}(\partial\tau)\big)},$$

$$E_{int}(u, \partial\tau) \;=\; \sum_{e \in \partial\tau} |curv(u)(e)| \;/\; L(\partial\tau),$$

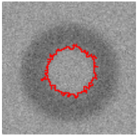
,

$$E_{con}(u, \partial\tau) \;=\; 1 \;/\; L(\partial\tau).$$

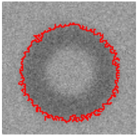
Application to object segmentation



Energy in a branch of the tree;
blue : our energy; green : snake energy.



(b)



(c)

Object detection principle

Significant minima $\Leftrightarrow$ Objects.

Application to object segmentation

Object detection strategy

Morphological closing in the shape-space: Get rid of the spurious minima.

Any attribute $\mathcal{A}$ can be used.

Generalization of MSER[Matas et al., BMVC, 2002]

stability functional τ : $\tau(\mathcal{N}_k) = (|\mathcal{N}_k^+| - |\mathcal{N}_k^-|)/|\mathcal{N}_k|$.

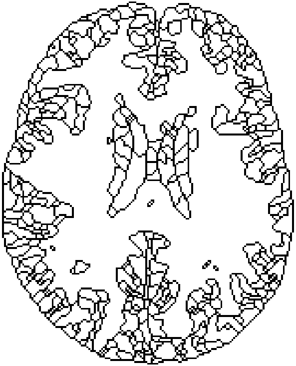
$|.|$: cardinality; $\mathcal{N}_k^+$ and $\mathcal{N}_k^-$: resp. ancestor and descendant of node $\mathcal{N}_k$ with a prefixed range of gray level compared with $\mathcal{N}_k$.

Minima of τ are spotted as interesting regions.

Generalization: Any tree $\mathcal{T}$, any attribute $\mathcal{A}$ can be used, and the morphological closing in shape-space filters the meaningless minima.

Saliency map

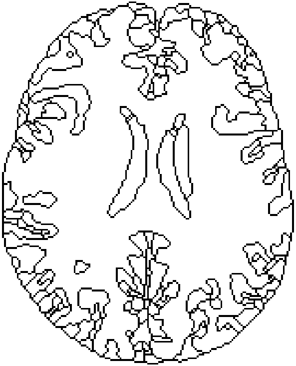
(a) Original image.



(b) Some contours.

Saliency map

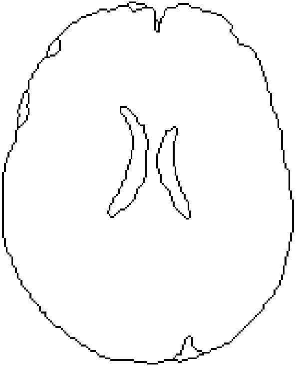
(a) Original image.



(b) Some contours.

Saliency map

(a) Original image.

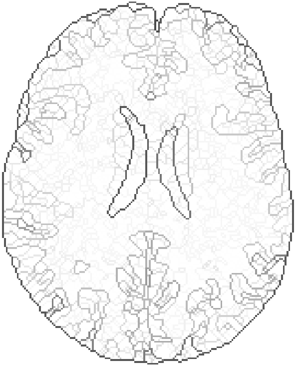


(b) Some contours.

Saliency map

Stacking the contours gives a saliency map [Najman & Schmitt, PAMI, 1996]

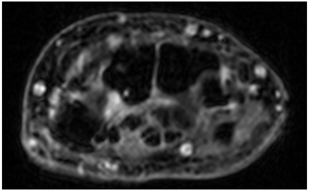
(a) Original image.



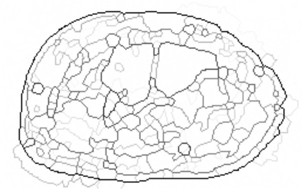
(b) A saliency map.

Different representations

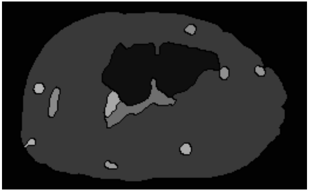
[L. Najman - JMIV - 2011] Mathematical definitions, equivalence between ultrametric watersheds, saliency maps and trees of segmentations



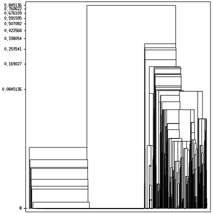
(a) Original image.



(b) Ultrametric watershed.



(c) One of the segmentations.



(d) Dendrogram.

Saliency maps from shape-based filterings

Idea

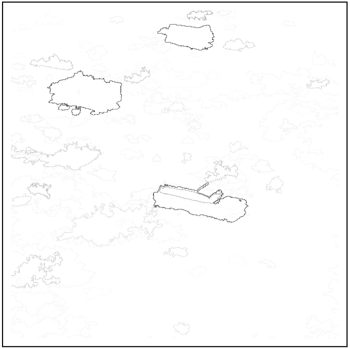
Extinction value for minima $\Leftrightarrow$ Persistence of objects $\overset{\mathcal{W}}{\Rightarrow}$ Saliency maps.

Strategy

$\mathcal{W}$: Weight the object contour with the maximum persistence of object that the contour belongs to.

Saliency maps from shape-based filterings

Input image.



Saliency map.

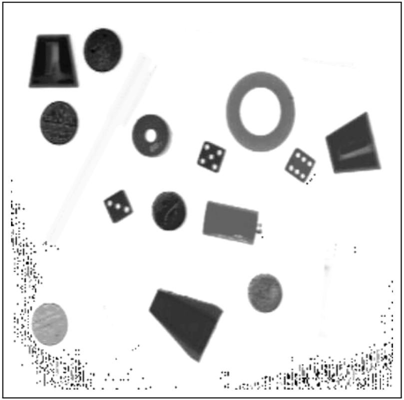
Outline

Shape-based lower/upper levelings

Input image.

Round objects based upper-leveling.

Shape-based lower/upper levelings



Difference of input image and the shape-based upper-leveling.

Blood vessels segmentation in retinal images

Important idea

1. *Use the green channel,*
2. *Black top-hat transform,*
3. *Extinction-based shape upper-leveling using circularity,*
4. *Preserved connected components are considered as blood vessels.*

Tested images

DRIVE database: Digital Retinal Images for Vessel Extraction.

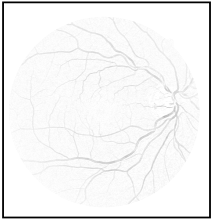
Performances measurements

1. Sensitivity and specificity : true positive and negative rate,
2. Accuracy: rate of pixels correctly classified,
3. kappa value: a statistical measure of inter-rater agreement.

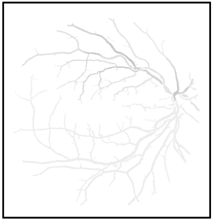
Blood vessels segmentation in retinal images

(a) Input color image.

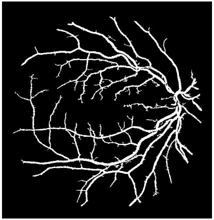
(b) Green channel.



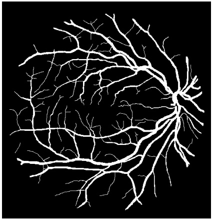
(c) Reversed black top-hat.



(d) Shape upper-leveling.



(e) Our segmentation.



(f) Manual segmentation.

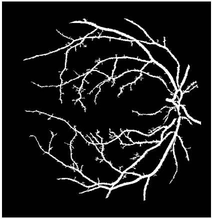
Blood vessels segmentation in retinal images

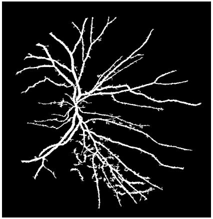

(a) Input image.

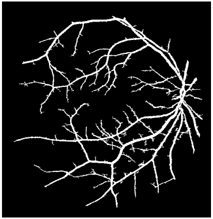

(b) Input image.

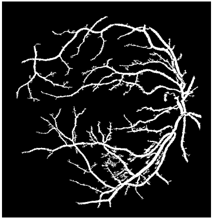

(c) Input image.


(d) Input image.


(e) Segmentation.


(f) Segmentation.


(g) Segmentation.


(h) Segmentation.

Blood vessels segmentation in retinal images

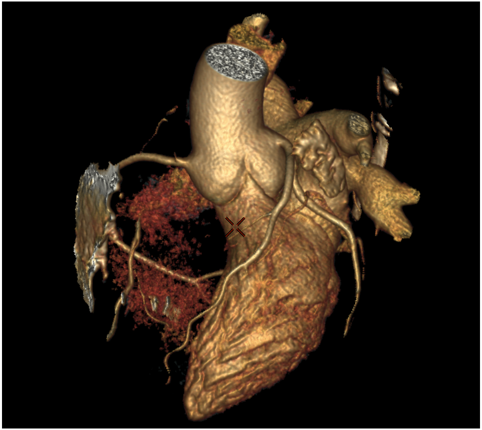
Method	Sensitivity	Specificity	Accuracy	Kappa
Expert	0.7761 (0.0593)	0.9725 (0.0082)	0.9473 (0.0048)	0.7589
Staal	0.7193 (0.0694)	0.9773 (0.0087)	0.9441 (0.0057)	0.7345
Niemeijer	0.6793 (0.0699)	0.9801 (0.0085)	0.9416 (0.0065)	0.7145
Zana	0.6696 (0.0764)	0.9769 (0.0079)	0.9377 (0.0078)	0.6971
Our method	**0.7613(0.0509)**	**0.9479(0.0237)**	**0.9231(0.0171)**	**0.6759**
Al-Diri			0.9258 (0.0126)	0.6716
Jiang	0.6478 (0.0642)	0.9625 (0.0130)	0.9222 (0.0070)	0.6399
Perez	0.7086 (0.1816)	0.9496 (0.0260)	0.9181 (0.0240)	0.6389
Chaudhuri	0.2716 (0.2119)	0.9794 (0.0388)	0.8894 (0.0231)	0.3357

Benchmark of different blood segmentation approaches.

Remark

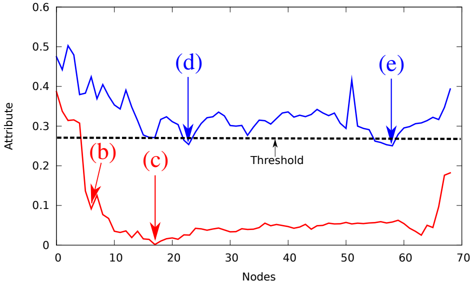
This is the result of only a “simple” filtering step.

It also works in 3D: Application to coronary arteries segmentation

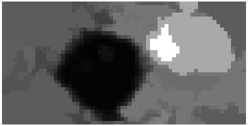


Path opening followed by elongation-based filtering

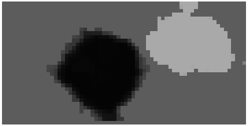
Morphological shapings



Evolution of circularity on two branches.

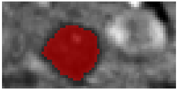


Thresholding.

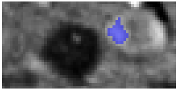


Our shaping.

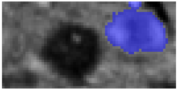
(b)



(c)

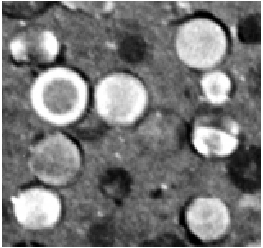


(d)

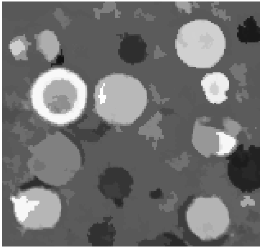


(e)

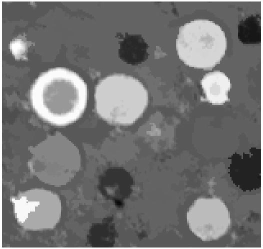
Morphological shapings



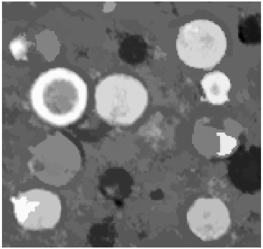
Input image.



Shaping based on $\mathcal{A}$

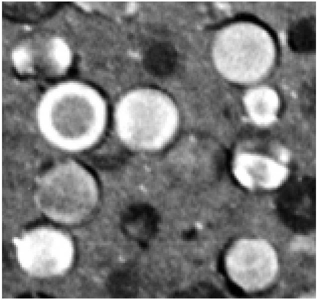


Low threshold of $\mathcal{A}$.

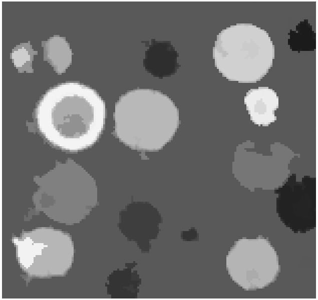


Higher threshold of $\mathcal{A}$.

Morphological shapings



Input image.



Our shaping 2.

Using a combination of attributes $\mathcal{A}$.

Object detection results

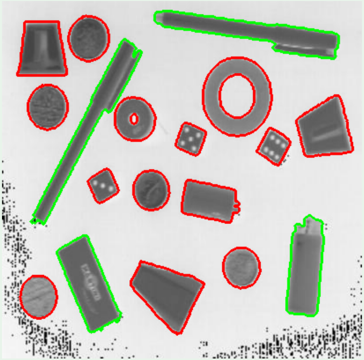
Context-based energy estimator

Input image.

Objects detected.

Object detection results

Shape attribute



Objects detected using shape attribute.
Red ones : circularity-based; Green ones : Inverse elongation-based.

Optic nerve head (ONH) segmentation

Important idea

1. *Use the red channel,*
2. *Classical morphological closing by a 2D disk,*
3. *Construct the tree of shapes and calculate a specific attribute using the fuzzy theory,*
4. *The best filling ellipse of the node having the minimal attribute is identified as the ONH.*

Tested images

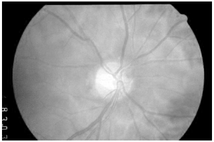
DRIONS database: Digital Retinal Images for Optic Nerve Segmentation Database.

Performances measurements

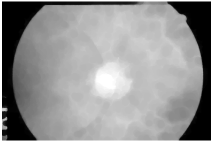
Discrepancy.

Optic nerve head (ONH) segmentation

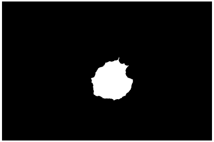
(a) Input color image.



(b) Red channel.

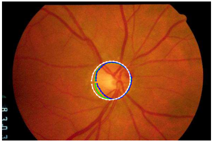


(c) Results of closing



(d) Detected CC.

(e) Segmented ONH.



(f) Manual results.

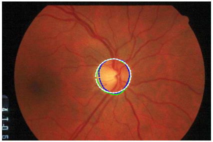
Optic nerve head (ONH) segmentation

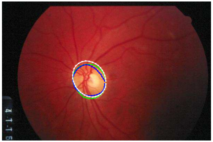

(a) Input image.

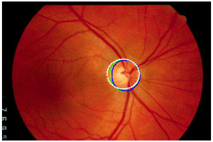

(b) Input image.

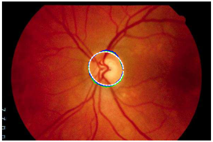

(c) Input image.


(d) Input image.


(e) ONHs.


(f) ONHs.


(g) ONHs.


(h) ONHs.

Optic nerve head (ONH) segmentation

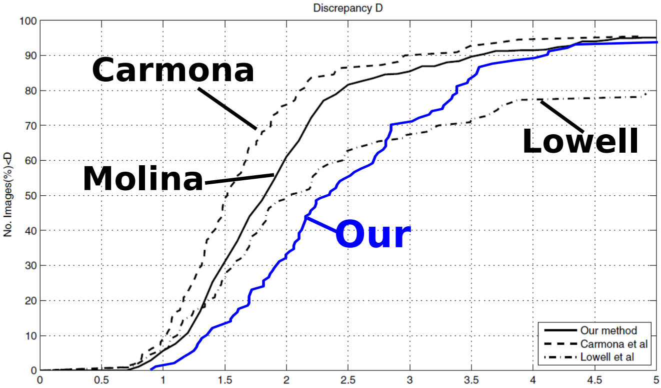
Carmona	96%
Molina	95%
Our	93.6%
Lowell	80%

Percentage of images whose discrepancy is fair

Remark

This is the result of only a “simple” filtering step.

Optic nerve head (ONH) segmentation



Accumulated discrepancy results for our detection method versus Carmona et al, Molina et al and Lowell et al.

Hierarchical simplification based on Mumford-Shah

Mumford-Shah energy with cartoon model

$$E_{\mathcal{T}} = \sum_{\partial\tau \in \mathcal{T}} \left(\sum_{p \in \mathcal{R}(\partial\tau)} \Big(u(p) - \overline{u}\big(\mathcal{R}(\partial\tau)\big) \Big)^2 + \nu L(\partial\tau) \right).$$

Attribute

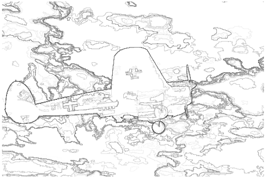
ν measures the simplification level.

Important idea

1. *Construct the tree of shapes,*
2. *Weight each node with the simplification level ν,*
3. *The saliency map yields a hierarchical simplification.*

Hierarchical simplification based on Mumford-Shah

Original.

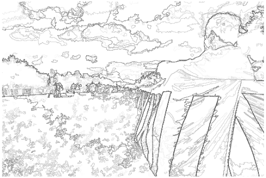


Saliency map.

Simplified.

Hierarchical simplification based on Mumford-Shah

Original.



Saliency map.

Simplified.

Hierarchical simplification based on Mumford-Shah

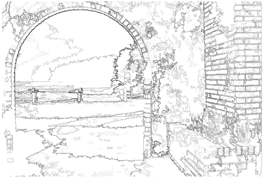
Original.

Saliency map.

Simplified.

Hierarchical simplification based on Mumford-Shah

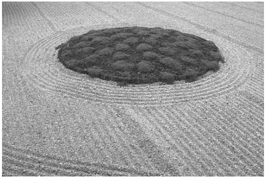
Original.



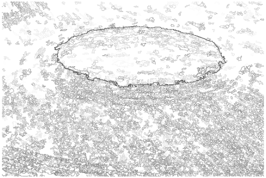
Saliency map.

Simplified.

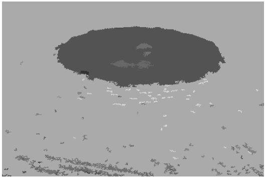
Hierarchical simplification based on Mumford-Shah



Original.



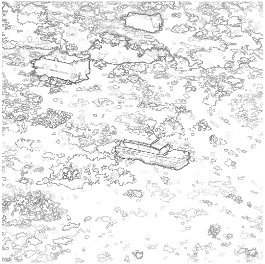
Saliency map.



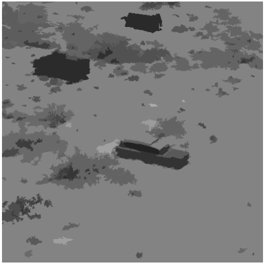
Simplified.

Hierarchical simplification based on Mumford-Shah

Original.



Saliency map.



Simplified.

Felzenswalb and Huttenlocher's algorithm

[Felzenswalb & Huttenlocher], IJCV, 2004

1. Compute a minimum spanning tree (MST) of a dissimilarity,
2. For each edge $\in$ MST linking two vertices x and y, in increasing order of their weights:
   (i) Find the region X that contains x,
   (ii) Find the region Y that contains y,
   (iii) Merge X and Y if

$$Diff(X, Y) < \min\{Int(X) + \frac{k}{|X|}, Int(Y) + \frac{k}{|Y|}\}.$$

Question

Is k a scale parameter?

Causality principle

- A contour present at a scale k_1 should be present at any scale $k_2 < k_1$.
- Not true with Felzenswalb and Huttenlocher's algorithm.

Original.

$k = 7500$ (8 regions).

$k = 9000$ (14 regions).

Application of our framework with attribute k

Answer

k is not a scale parameter.

Attribute from k

$$k = \max\left\{\left(Diff(X,Y) - Int(X)\right) \times |X|, \left(Diff(X,Y) - Int(Y)\right) \times |Y|\right\}.$$

Hierarchical image segmentation on BSDS500

Important idea

1. *Calculate the distance between neighboring pixels,*
2. *Construct a minimum spanning tree (MST),*
3. *Compute attribute k,*
4. *The saliency map yields an hierarchical image segmentation.*

Tested images

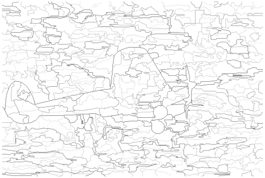
BSDS500: Berkeley Segmentation Data Set and Benchmarks 500.

Performance measurements

1. Ground-truth Covering [Arbeláez et al., PAMI, 2011],
2. Probabilistic Rand Index [Arbeláez et al., PAMI, 2011].

Hierarchical image segmentation on BSDS500

Original.



Saliency map.

Segmentation(11 regions).

Hierarchical image segmentation on BSDS500

Original.

Saliency map.

Segmentation(70 regions).

Hierarchical image segmentation on BSDS500

Original.

Saliency map.

Segmentation(20 regions).

Hierarchical image segmentation on BSDS500

Caution!

Preliminary results

Benchmarks

Our method obtains better results than the results of method of FH, and of method of Guimarães for optimal dataset scale (ODS), and for optimal image scale (OIS).

Method	GT Covering			Prob. Rand. Index	
	ODS	OIS	Best	ODS	OIS
FH	0.43	0.53	0.68	0.76	0.79
Guimarães	0.46	0.53	0.60	0.76	0.81
Ours	**0.50**	**0.57**	**0.66**	**0.77**	**0.82**

Comparison of the hierarchical segmentation obtained with Felzenswalb and Huttenlocher's algorithm, method of Guimarães et al., and our method.

Outline

Conclusion

- Object filtering

1. Encompass the state of art,
2. Shape-based lower/upper-levelings,
3. Morphological shapings.

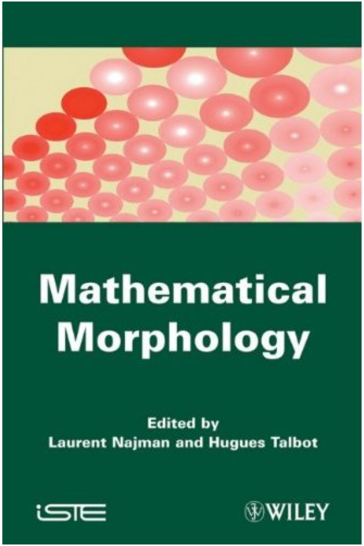
- Object detection

1. Context-based estimator,
2. Specific attribute $\mathcal{A}$ for ONH segmentation,
3. Saliency map.

Perspectives

- Attributes $\mathcal{A}$ and $\mathcal{AA}$,
- **Learning of the attributes**,
- Strategies of dealing with second tree $\mathcal{TT}$,
- More Properties of the morphological shapings,
- Saliency maps.

Thank for your attention !



Pink: http://pinkhq.com
Olena: http://www.lrde.epita.fr/cgi-bin/twiki/view/Olena