Recognizing hierarchical watersheds

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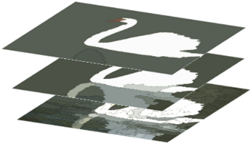
Hierarchical watersheds

- Sequences of nested partitions such that each partition is a watershed of a filtering of the initial relief/map according to regional attributes, *e.g.*, on geometric, photometric, or learned, information
- Good performance on natural images [P18]
- Hierarchies of segmentation can be equivalently represented by saliency maps

Original image

Hierarchical watershed $\mathcal{H}$



Saliency map of $\mathcal{H}$

[P18] B. Perret, J Cousty, S. Guimaraes, D. Maia. Evaluation of hierarchical watersheds. IEEE TIP. 2018.

Combination of hierarchies (of watersheds)

- Based on saliency maps combination [C17, M17]
  - with, *e.g.*, infimum, supremum, or linear combination

[C17] J. Cousty, L. Najman, Y. Kenmochi, S. Guimarães. Hierarchical segmentations with graphs: quasi-flat zones, minimum spanning trees, and saliency maps. JMIV. 2017

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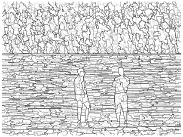
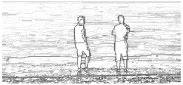
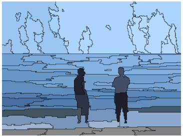
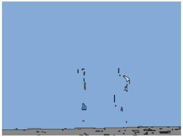


Area attribute



Dynamics attribute





One level of each hierarchy with 75 regions

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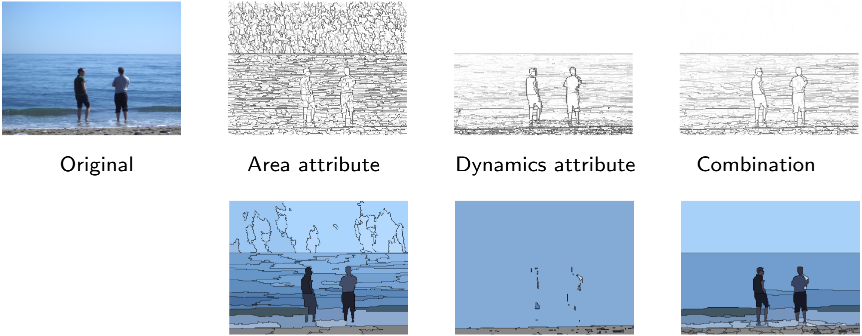

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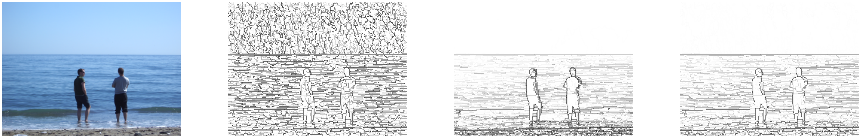
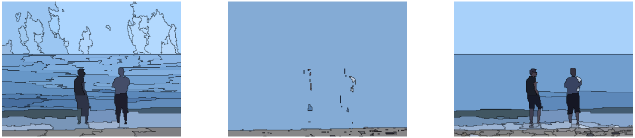


Original | Area attribute | Dynamics attribute | Combination



One level of each hierarchy with 75 regions

Does the combinations of hierarchical watersheds always result in a hierarchical watershed?

Hierarchical segmentation

Hierarchical segmentation technique proposed in [A11]:

[A11] P. Arbelaez, M. Maire, C. Fowlkes and J. Malik. Contour detection and hierarchical image segmentation. IEEE transactions on pattern analysis and machine intelligence. 2011

Hierarchical segmentation

Hierarchical segmentation technique proposed in [A11]:

- Contour-detection with multiscale cue combination (*m*Pb)

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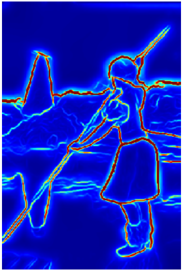

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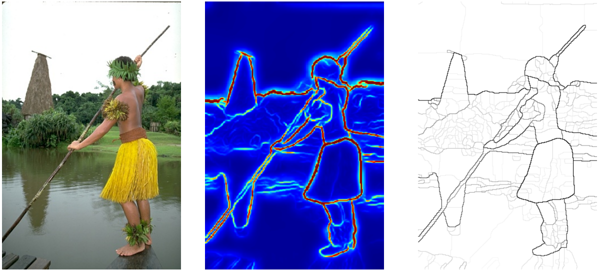
- Hierarchical segmentation based on the Oriented Watershed Transform (OWT)



I $\quad$ mPb $\quad$ $\mathcal{H}$

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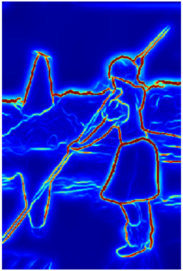

*m*Pb

$\mathcal{H}$

Is $\mathcal{H}$ a hierarchical watershed of *mPb*?

Recognition of hierarchical watersheds

Problem

Given a hierarchy $\mathcal{H}$ and a weighted graph (G, w):

- *Recognize if $\mathcal{H}$ is a hierarchical watershed of (G, w)*
- **Naive approach:** *factorial time complexity*

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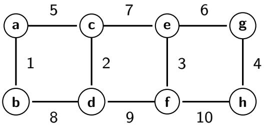
Contributions

- *Characterization of hierarchical watersheds*
- *Quasi-linear algorithm to recognize the hierarchical watersheds*

Outlines

Graph settings

- (G, w) is an edge-weighted graph
  - which can be a pixel-adjacency graph
  - edge weight can represent a gradient of intensity



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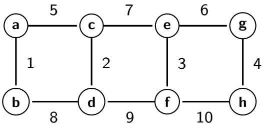

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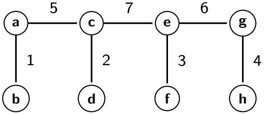

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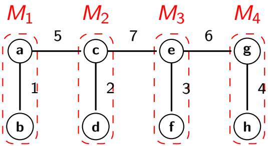
- For simplification, we consider that *the edges of G has pairwise distinct weights for w, which implies that (G, w) has a single MST*
- $\mathcal{M} = (M_0, \ldots, M_\ell)$ is any sequence of the regional minima of w ranked by importance according to some given attribute



Hierarchical watersheds

- A *hierarchical watershed of* (G, w) *for* $\mathcal{M}$ is a hierarchy of partitions $(\mathbf{P}_0, \ldots, \mathbf{P}_\ell)$ such that, for any $i \in \{0, \ldots, \ell\}$:
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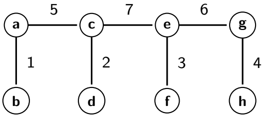

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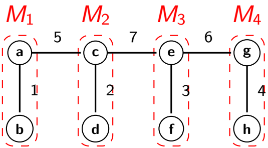

Minima of w

$$\mathcal{M} = (M_1, M_2, M_3, M_4)$$

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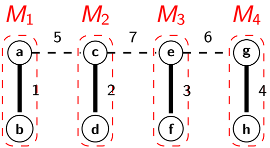

$\{a, b\}$ $\{c, d\}$ $\{e, f\}$ $\{g, h\}$ $\mathbf{P}_0$

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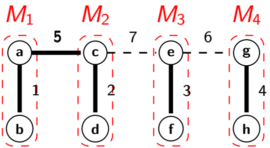

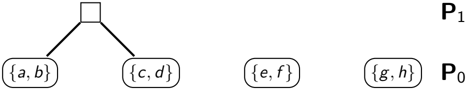

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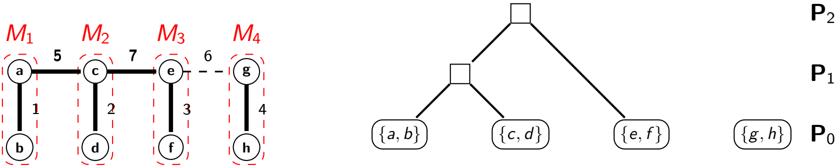

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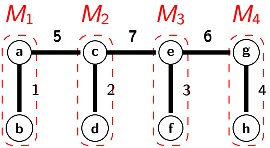

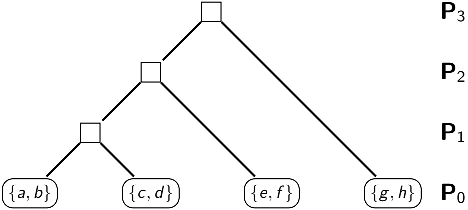

$(\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3)$:
hierarchical watershed of (G, w) by $\mathcal{M} = (M_1, M_2, M_3, M_4)$

Hierarchical watersheds

Definition

We say that $\mathcal{H}$ is a *hierarchical watershed of* (G, w) if there is a sequence $\mathcal{M}$ of minima such that $\mathcal{H}$ is the hierarchical watershed of (G, w) for $\mathcal{M}$

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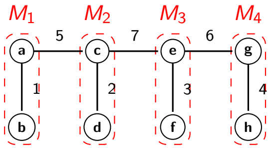
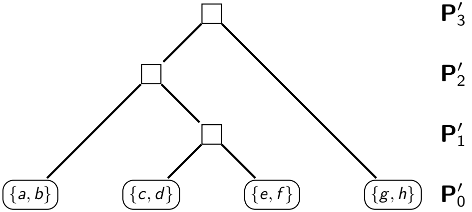
Remark: there are hierarchies which are not hierarchical watersheds of (G, w), e.g., $(\mathbf{P}'_0, \mathbf{P}'_1, \mathbf{P}'_2, \mathbf{P}'_3)$ *is* **not** *a hierarchical watershed of* (G, w)



(G, w)



$\mathcal{H}'$

Outlines

Characterization of hierarchical watersheds

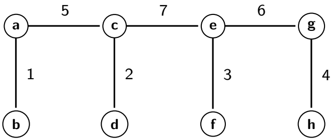
Key notions to present the characterization of hierarchical watersheds (Theorem 3):

- Binary partition hierarchy by altitude ordering (single linkage clustering with connectivity constraint) [C13]
- One-side increasing map

[C13] J. Cousty, L. Najman, B. Perret. Constructive links between some morphological hierarchies on edge-weighted graphs. ISMM 2013.

Binary partition hierarchy

Characterization of HW:
⟹ 1. binary partition hierarchy
2. one-side increasing map
3. Theorem 3



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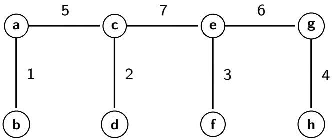

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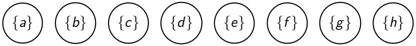

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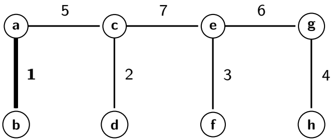

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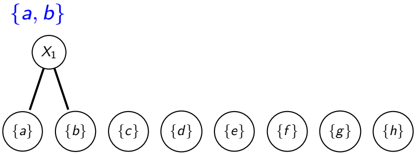

Any non-leaf region R of $\mathcal{B}$ can be mapped to an edge u_R of G which is then called the building edge of R (shown in blue)

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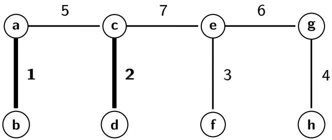

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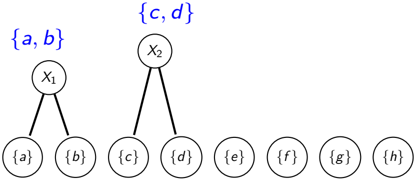

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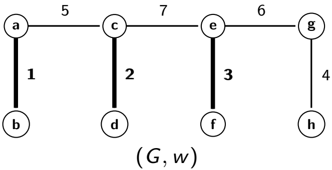

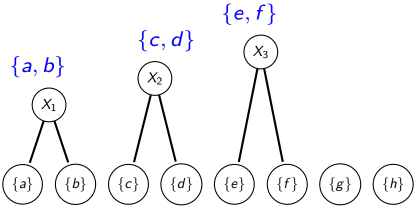

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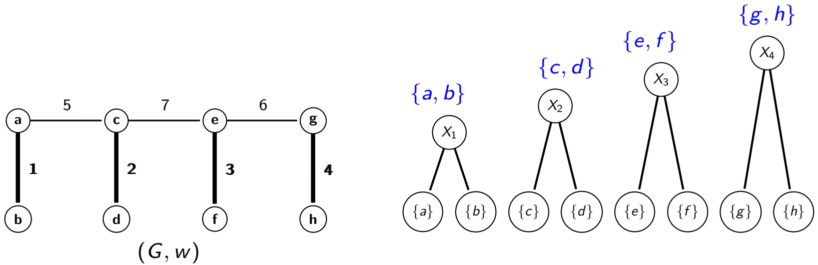

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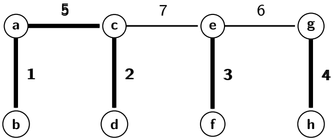
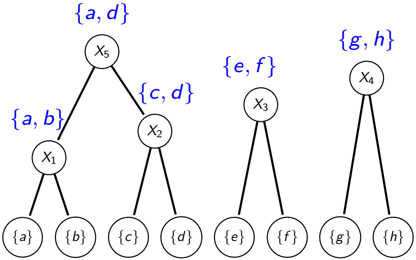


(G, w)



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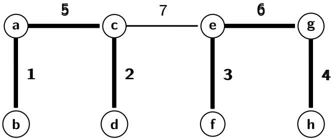

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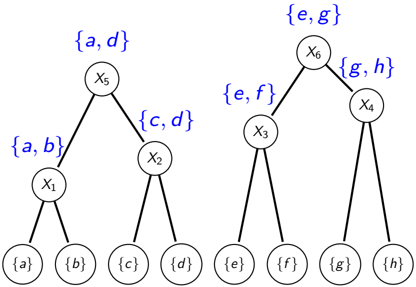

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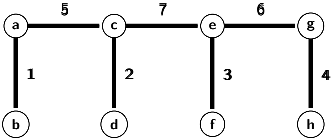
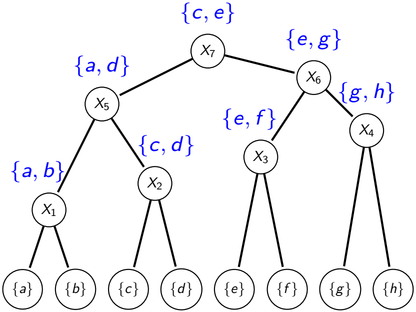


(G, w)



$\mathcal{B}$

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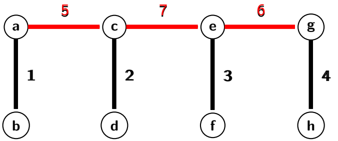

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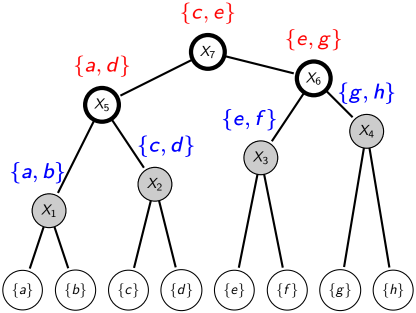

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Minima (in grey) and watershed-cut edges (in red)

One-side increasing map (intuitive idea)

In order for a hierarchy $\mathcal{H}$ to be a hierarchical watershed of a weighted graph (G, w), we need:

- The finest level of $\mathcal{H}$ to be the watershed segmentation of (G, w);
- Exactly one pair of regions to be merged at each level of the hierarchy; and
- Any region to be first merged to their "most similar" neighbors before being merged to their "least similar" neighbors.

One-side increasing map

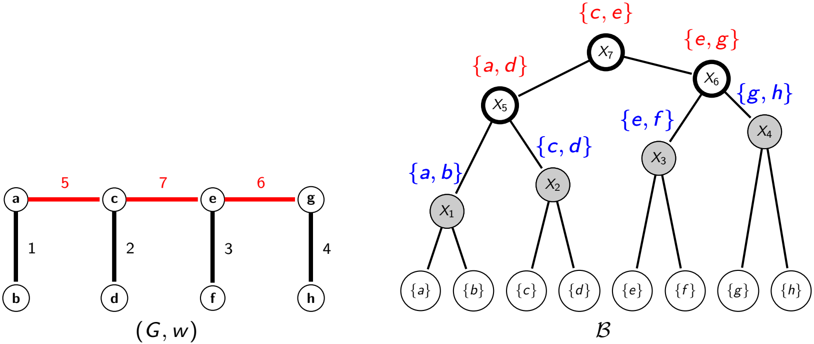
Definition (one-side increasing map)

Given a weighted graph (G, w), let $\mathcal{B}$ be the binary partition hierarchy of w. We say that a map f from E into $\mathbb{R}^+$ is a *one-side increasing map for* $\mathcal{B}$ if:

1. $range(f) = \{0, \ldots, n-1\}$;
2. for any u in E, $f(u) > 0$ if and only if u is a watershed-cut edge of w; and
3. for any u in E, there exists a child R of R_u such that $f(u) \geq \vee\{f(v)$ such that R_v is included in $R\}$, where $\vee\{\} = 0$.

One-side increasing map

Characterization of HW:
1. binary partition hierarchy
⟹ 2. one-side increasing map
3. Theorem 3



Minima (in grey) and watershed-cut edges (in red)

One-side increasing map (example)

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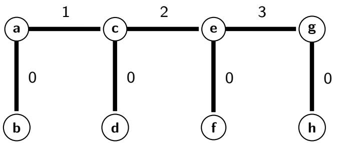

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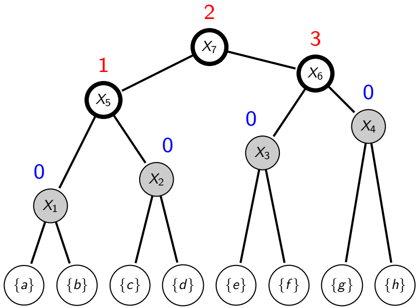

$\mathcal{B}$ and f

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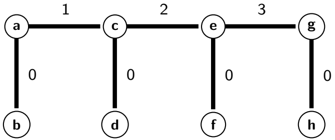

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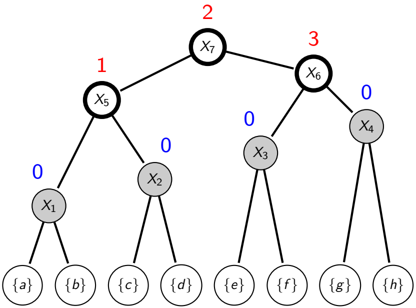

$\mathcal{B}$ and f

$range(f) = \{0, \ldots, 3\}$?

One-side increasing map (example)

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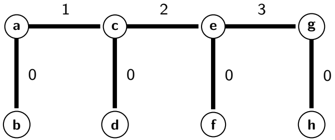

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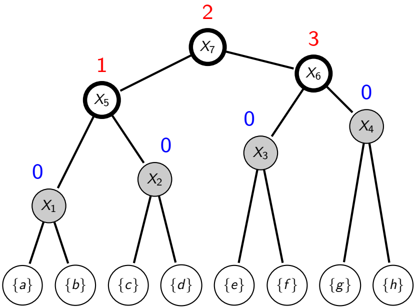

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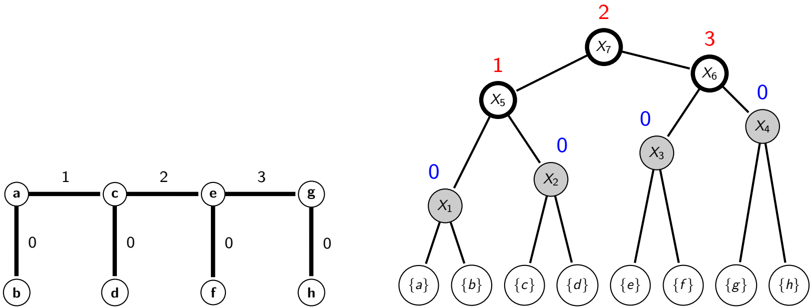

f $\quad$ $\mathcal{B}$ and f

$f(u) > 0$ if and only if u is a watershed-cut edge of w?

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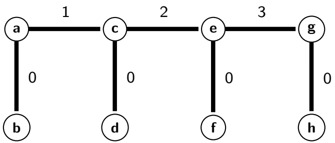

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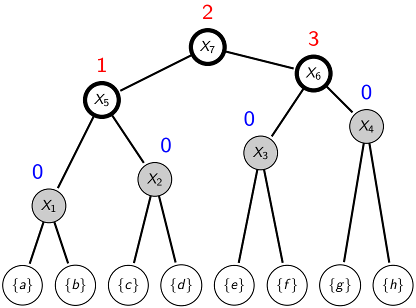

$\mathcal{B}$ and f

$f(u) > 0$ if and only if u is a watershed-cut edge of w ✓

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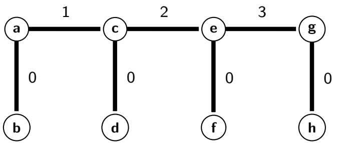

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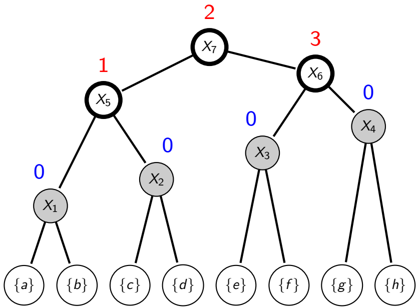

$\mathcal{B}$ and f

for any u in E, there exists a child R of R_u such that $f(u) \geq \vee\{f(v)$ such that R_v is included in $R\}$?

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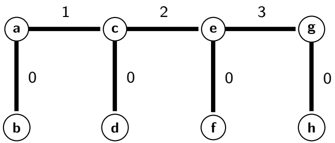

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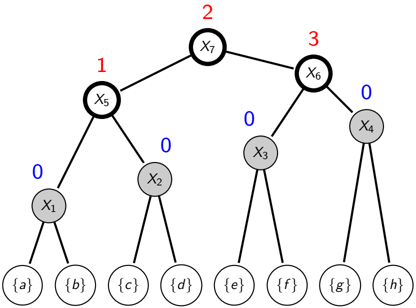

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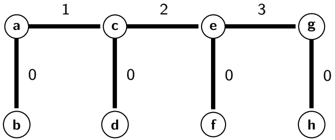

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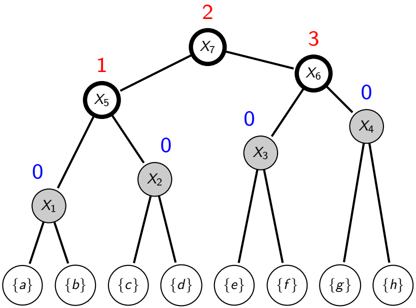

$\mathcal{B}$ and f

Therefore, f is a one-side increasing map for $\mathcal{B}$

One-side increasing map (counter-example)

Characterization of HW:
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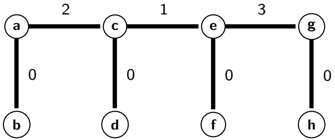

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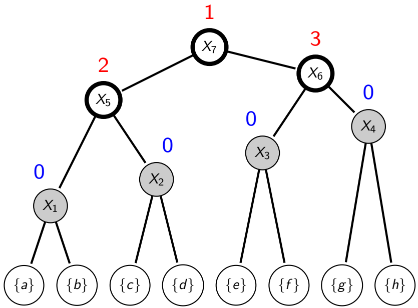

$\mathcal{B}$ and s

Minima (in grey) and watershed-cut edges (in red)

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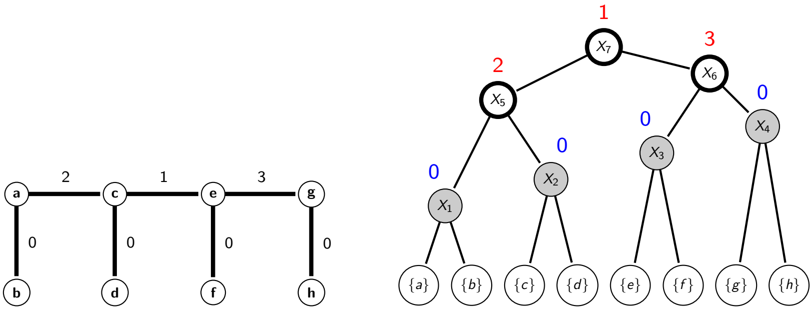

s $\qquad\qquad$ $\mathcal{B}$ and s

$$range(s) = \{0, \ldots, 3\}?$$

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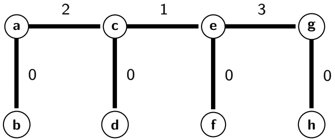

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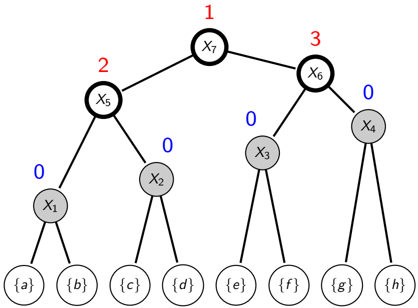

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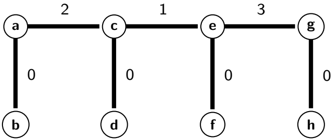

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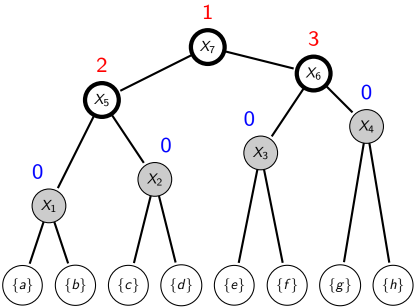

$\mathcal{B}$ and s

$s(u) > 0$ if and only if u is a watershed-cut edge of w?

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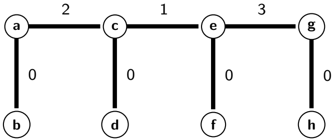

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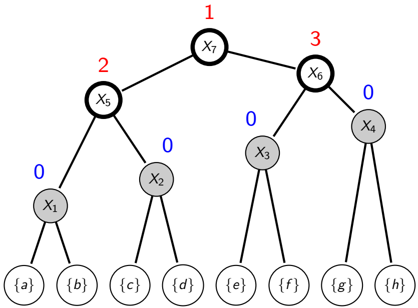

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$s(u) > 0$ if and only if u is a watershed-cut edge of w ✓

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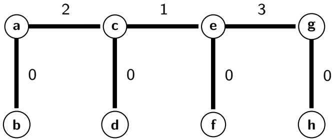

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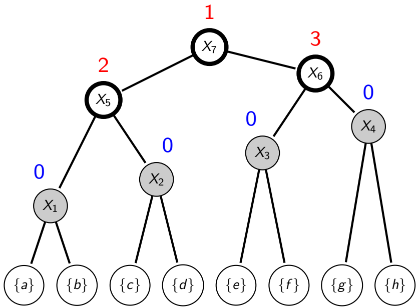

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for any u in E, there exists a child R of R_u such that $s(u) \geq \vee\{s(v)$ such that R_v is included in $R\}$?

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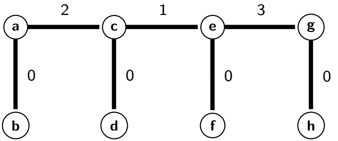

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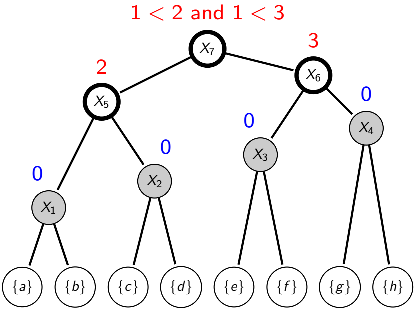

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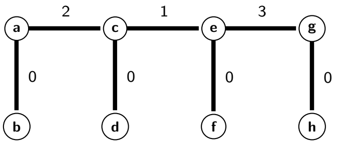

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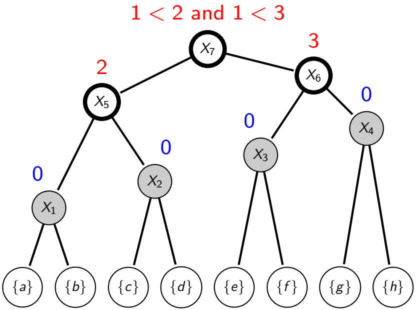

$\mathcal{B}$ and s

Therefore, s is not a one-side increasing map for $\mathcal{B}$

Characterization of hierarchical watersheds

Theorem (characterization of hierarchical watersheds)

- *Let* (G, w) *be a weighted graph and let* $\mathcal{B}$ *be the binary partition hierarchy of* w*. Let* $\mathcal{H}$ *be a hierarchy and let* $\Phi(\mathcal{H})$ *be the saliency map of* $\mathcal{H}$*.*
- *The hierarchy* $\mathcal{H}$ *is a hierarchical watershed of* (G, w) *if and only if* $\Phi(\mathcal{H})$ *is a one-side increasing map for* $\mathcal{B}$*.*

Characterization of hierarchical watersheds

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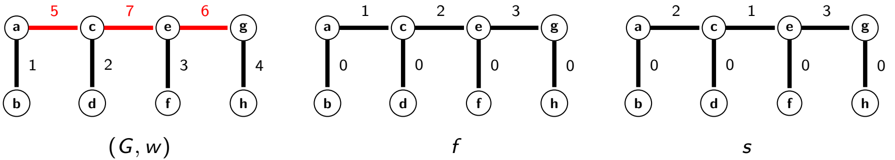

Then, f **is** the saliency of a hierarchical watershed of (G, w), but s **is not**.

Outlines

1. Hierarchical watersheds
2. Characterization of hierarchical watersheds
3. Algorithm to recognize hierarchical watersheds
4. Conclusion and perspectives

Algorithm to recognize hierarchical watersheds

We determine if a map f is the saliency map of a hierarchical watershed of (G, w) through the following steps:

1. Compute the binary partition hierarchy $\mathcal{B}$ of w
2. Compute the set $WS(w)$ of watershed-cut edges of w
3. Compute the number n of minima of w
4. For each edge u of G, compute the value $Max[u]$ which corresponds to $\vee\{f(v) \mid R_u \subseteq R_u\}$
5. For each edge u of G:
    1. If $f(u)$ not in $\{0, \ldots, n-1\}$, then return **false**
    2. If u is not in $WS(w)$ and $f(u) \neq 0$, then return **false**
    3. If u is in $WS(w)$ and if $f(u)$ is not unique, then return **false**
    4. If u is in $WS(w)$ and if $Max[u] < Max[v_1]$ and $Max[u] < Max[v_2]$, then return **false**
6. return **true**

Algorithm to recognize hierarchical watersheds

Complexity analysis:

- As shown in [N13], the binary partition hierarchy $\mathcal{B}$ of (G, w) can be computed in **quasi-linear time** with respect to the number of edges of G
- Then, given the hierarchy $\mathcal{B}$, the minima and the watershed-cut edges of (G, w) can be obtained in **linear time**
- The array *Max* can be also obtained in **linear time** if computed from the leaves to the root of $\mathcal{B}$
- Finally, the three conditions for f to the a one-side increasing map of $\mathcal{B}$ can be verified in **linear time** as well
- *Therefore, the proposed algorithm has a* **quasi-linear time** *complexity*

[N13] L. Najman, J. Cousty, B. Perret. Playing with Kruskal: algorithms for morphological trees in edge-weighted graphs.. ISMM 2013.

Outlines

Conclusion and perspectives

Summary

- *Characterization of hierarchical watersheds through binary partition hierarchies*
- *Quasi-linear time algorithm to recognize hierarchical watersheds*

Perspectives

- *Answer the question: does the combination of hierarchical watersheds always result in a hierarchical watershed?*
- *Extension to arbitrary weighted graphs*
- Waterhseding operator *that converts any hierarchy into a hierarchical watershed (ISMM2019)*