Watersheding hierarchies: recognition of hierarchical watersheds

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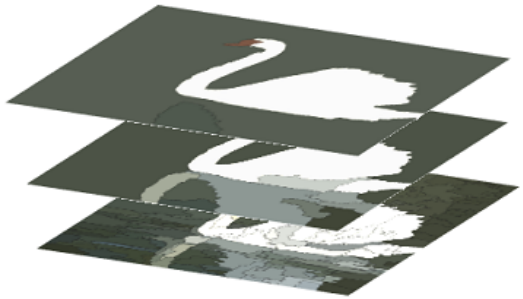
Hierarchical watersheds

- Sequences of nested partitions such that:
  - Each partition is a watershed of a filtering of the initial relief/map
  - Filtering according to regional attributes based, *e.g.*, on geometric, photometric, or learned, information
  - Good performance on natural images [P18]

Original image

Hierarchical watershed $\mathcal{H}$



Saliency map of $\mathcal{H}$ [1]

[1] *i.e.*, a characteristic function of $\mathcal{H}$, see previous presentation of Jean Cousty

[P18] B. Perret, J Cousty, S. Guimaraes, D. Maia. Evaluation of hierarchical watersheds. IEEE TIP. 2018.

Combination of hierarchies (of watersheds)

- Based on saliency maps combination [C17, M17]
  - with, *e.g.*, infimum, supremum, or linear combination

[C17] J. Cousty, L. Najman, Y. Kenmochi, S. Guimarães. Hierarchical segmentations with graphs: quasi-flat zones, minimum spanning trees, and saliency maps. JMIV. 2017

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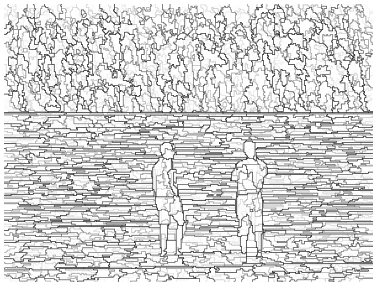
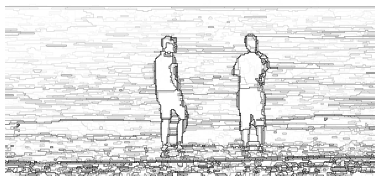
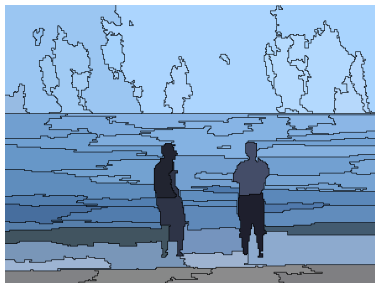
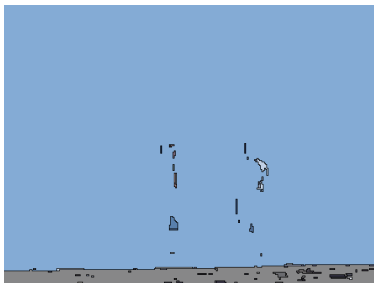


Area attribute



Dynamics attribute





One level of each hierarchy with 75 levels

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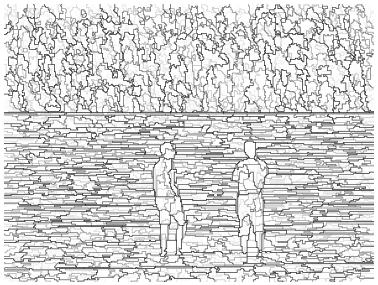

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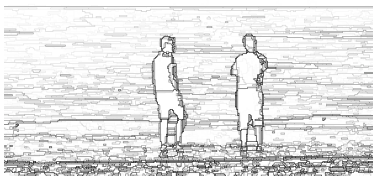

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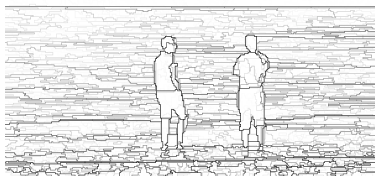
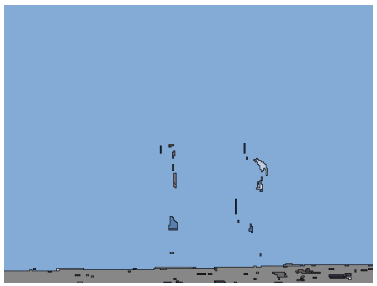
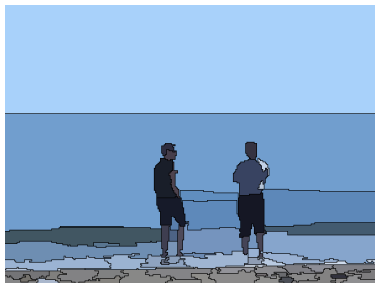


Combination by average





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Combinations of hierarchical watersheds

Theoretical problem

Does the combination of two hierarchical watersheds always result in a hierarchy which itself is a hierarchical watershed?

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- Is there a regional attribute A such that the combination of two hierarchical watersheds is the hierarchical watershed obtained when filtering the function with attribute A?
- Does the combination of hierarchical watersheds allow to go beyond hierarchical watersheds?

Recognition of hierarchical watersheds

Problem

Given a hierarchy $\mathcal{H}$ and a weighted graph (G, w):

- *Recognize if $\mathcal{H}$ is a hierarchical watershed of (G, w)*

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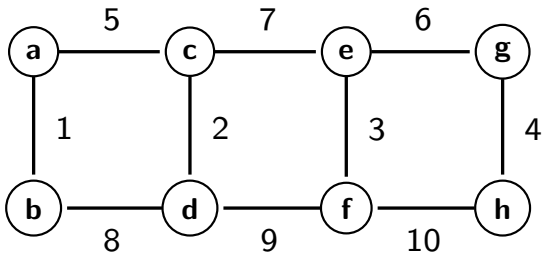
Contributions

- *Introduction of the* watersheding transform
  - *which transforms any hierarchy into a hierarchical watershed*
  - *whose fixed points are the hierarchical watersheds*
- *Characterization of hierarchical watersheds*
- *Quasi-linear algorithm to recognize the hierarchical watersheds*

Outlines

Graph settings

- (G, w) is an edge-weighted graph
  - which can be a pixel-adjacency graph
  - edge weight can represent a gradient of intensity



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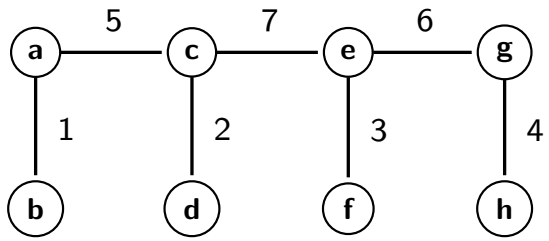

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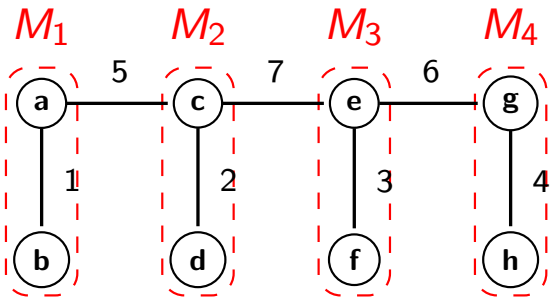
  - edge weight can represent a gradient of intensity
- For simplification, we consider that *G is a tree and that the edge-weights are pairwise distinct*
- $\mathcal{M} = (M_0, \ldots, M_\ell)$ is any sequence of the regional minima of w ranked by importance according to some given attribute



Hierarchical watersheds

- A *hierarchical watershed of* (G, w) *for* $\mathcal{M}$ is a hierarchy of partitions $(\mathbf{P}_0, \ldots, \mathbf{P}_\ell)$ such that, for any $i \in \{0, \ldots, \ell\}$:
  - $\mathbf{P}_i$ is the connected component partition of a minimum spanning forest rooted in the minima ranked after i

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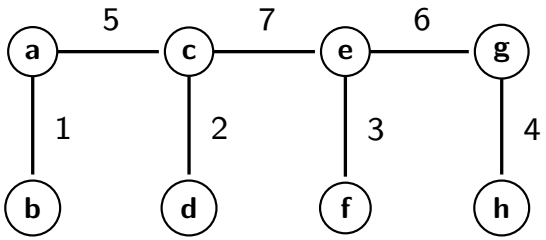

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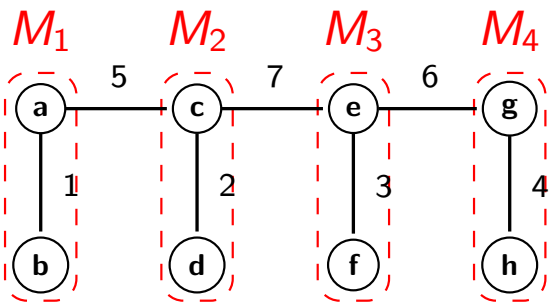

Minima of w

$$\mathcal{M} = (M_1, M_2, M_3, M_4)$$

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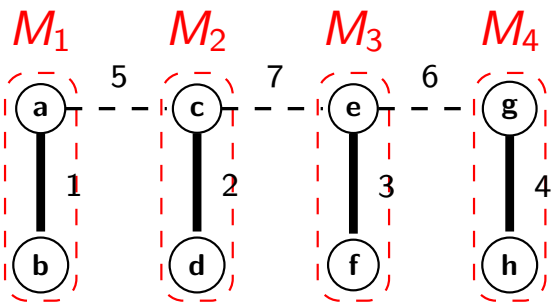

$\{a, b\}$ $\{c, d\}$ $\{e, f\}$ $\{g, h\}$ $\mathbf{P}_0$

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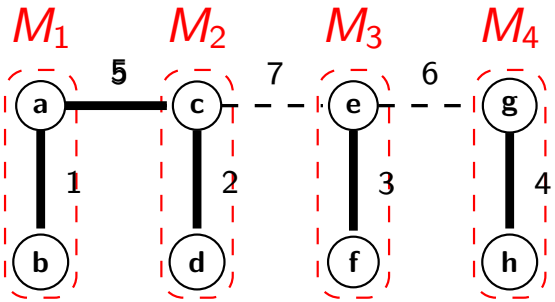

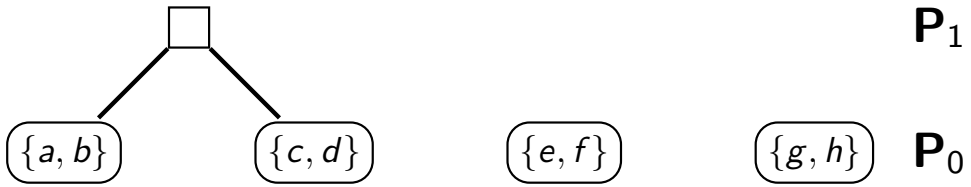

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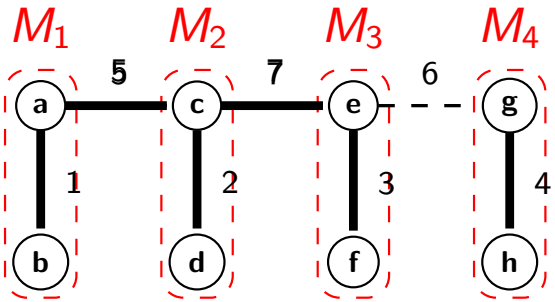

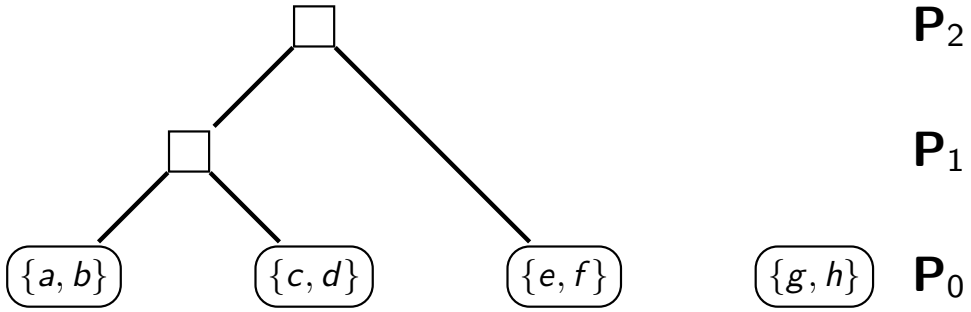

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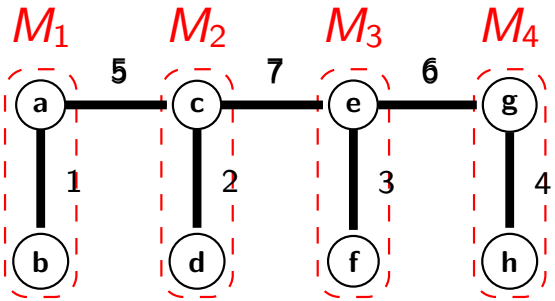

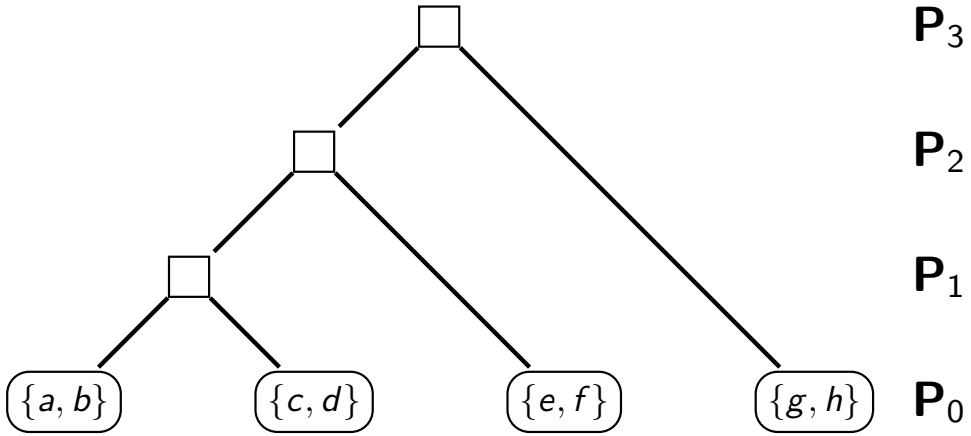

$(\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3)$:
hierarchical watershed of (G, w) for $\mathcal{M}$

Hierarchical watersheds

Definition

We say that $\mathcal{H}$ is a *hierarchical watershed of* (G, w) if there is a sequence $\mathcal{M}$ of minima such that $\mathcal{H}$ is the hierarchical watershed of (G, w) for $\mathcal{M}$

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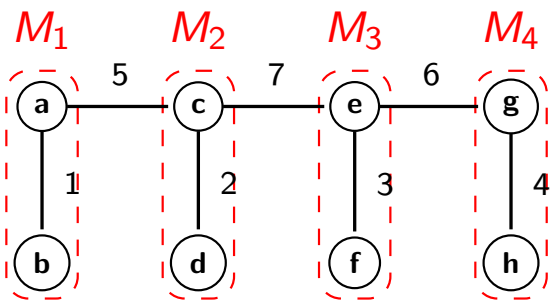
Remark: there are hierarchies which are not hierarchical watershed of a graph, e.g., $(\mathbf{P}'_0, \mathbf{P}'_1, \mathbf{P}'_2, \mathbf{P}'_3)$ *is* **not** *a hierarchical watershed of* (G, w)



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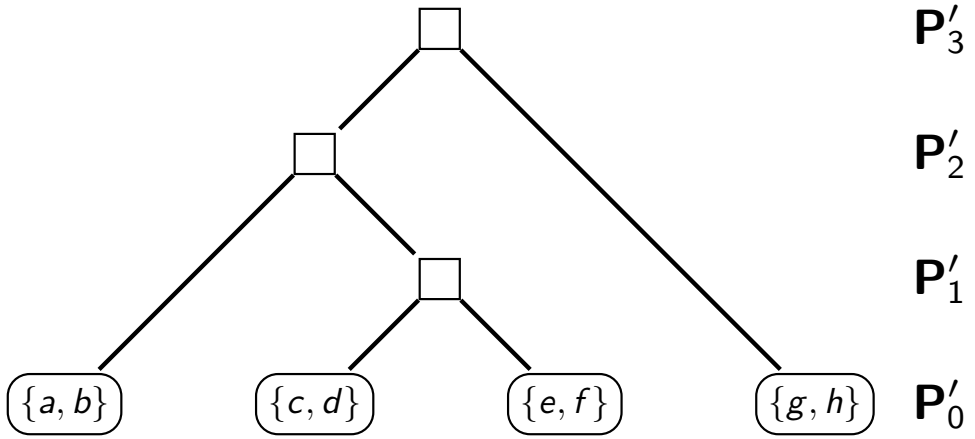

Outlines

Hierarchical watershed recognition: general overview

Key idea for recognizing hierarchical watersheds:

- "Inverse" the hierarchical watershed algorithm of [C13, N13]

Given a hierarchy $\mathcal{H}$ and a weighted graph (G, w):

1. compute the *binary partition hierarchy (by altitude ordering)* of w
2. compute the saliency map s of $\mathcal{H}$
3. compute the *watershed-cut ordering* s_o from s
4. compute the *extinction estimation* ξ from s_o
5. compute the *persistence estimation* π from ξ
6. if $\pi = s$ then and only the $\mathcal{H}$ is a a hierarchical watershed

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Binary partition hierarchy

Recognition algorithm:
1. binary partition hierarchy $\mathcal{B}$
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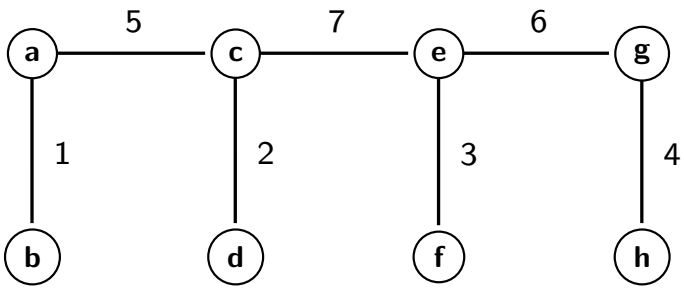
6. verify if $\pi_s = s$



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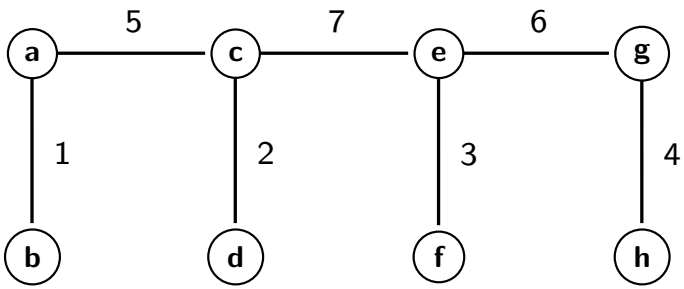

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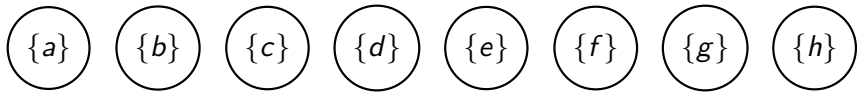

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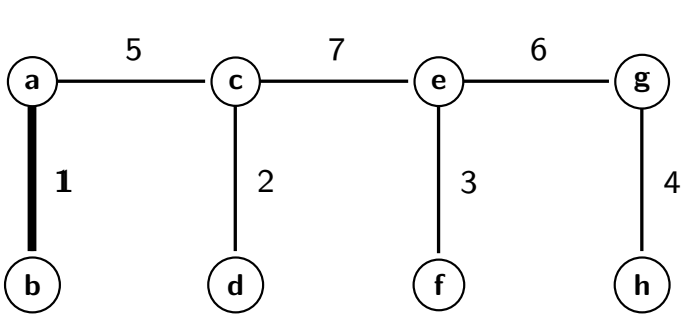

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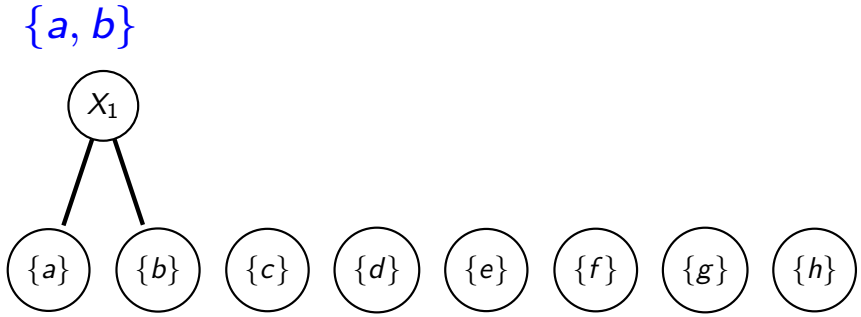

Any non-leaf region R of $\mathcal{B}$ can be mapped to an edge u_R of G which is then called the building edge of R (shown in blue)

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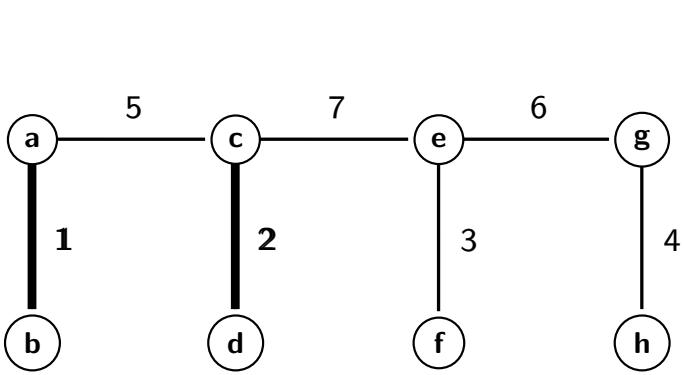

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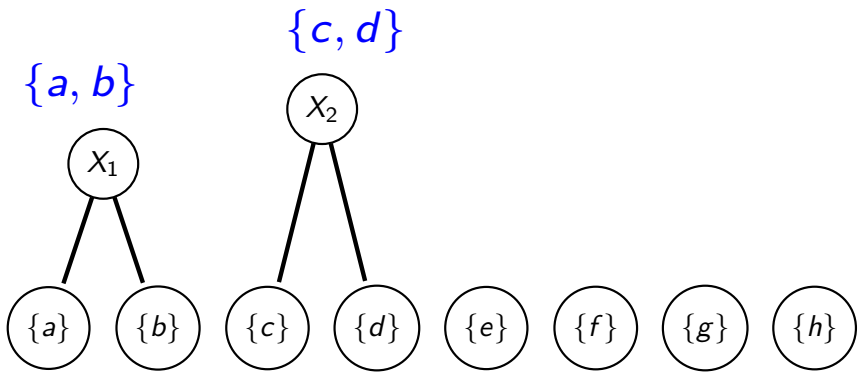

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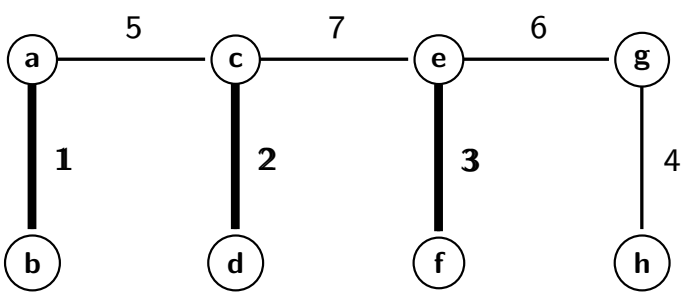

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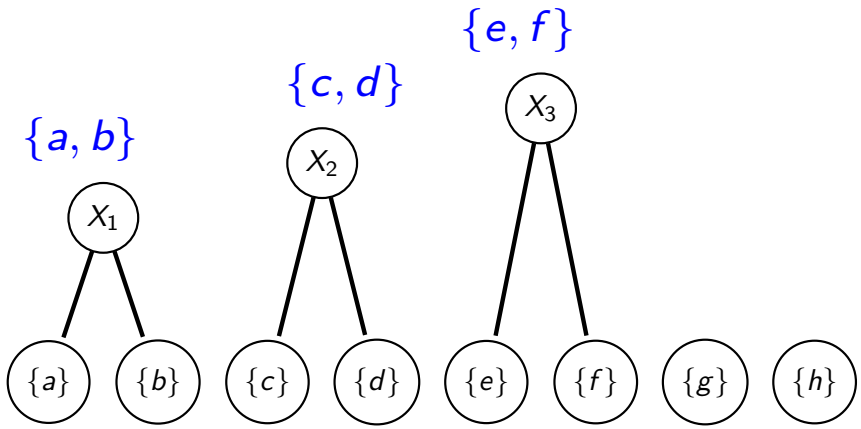

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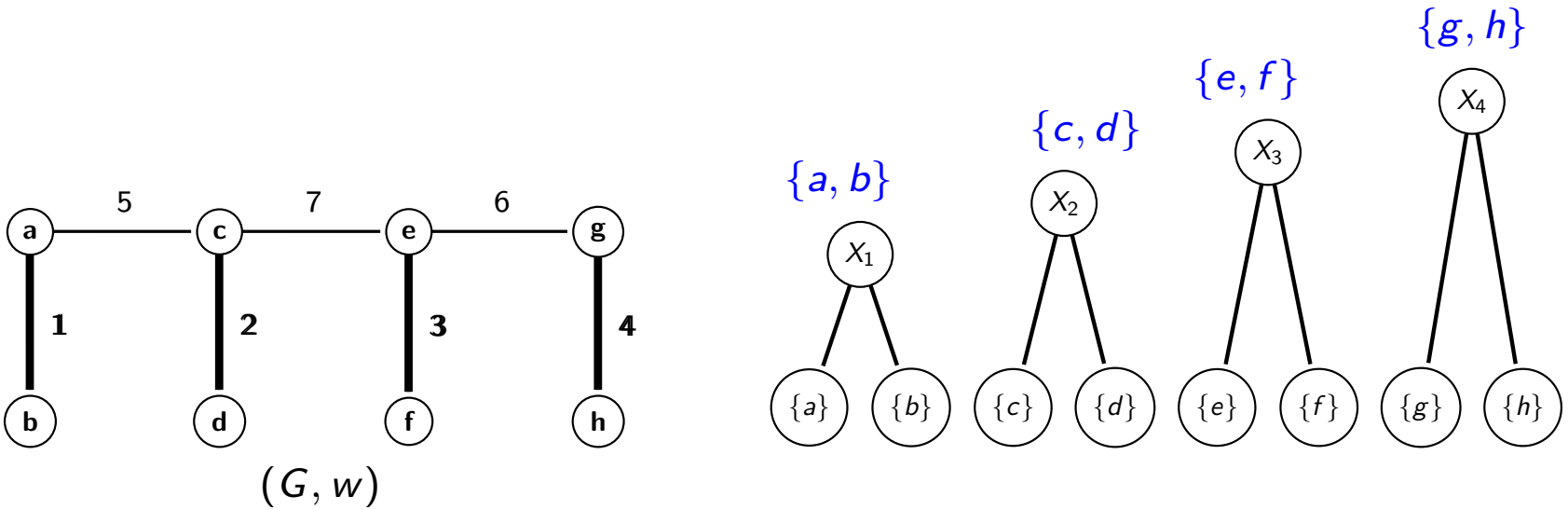

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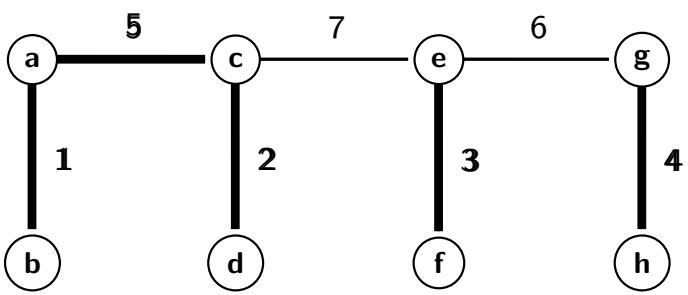
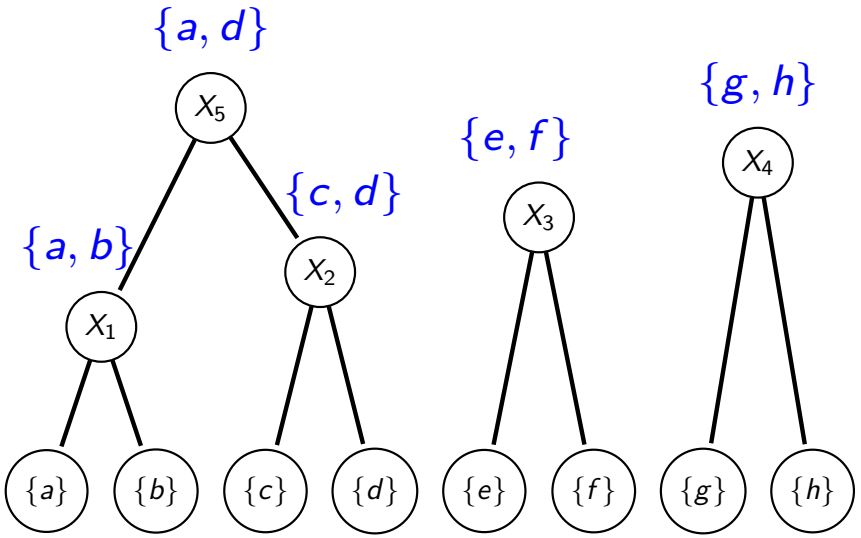


(G, w)



Any non-leaf region R of $\mathcal{B}$ can be mapped to an edge u_R of G which is then called the building edge of R (shown in blue)

Binary partition hierarchy

Recognition algorithm:
1. binary partition hierarchy $\mathcal{B}$
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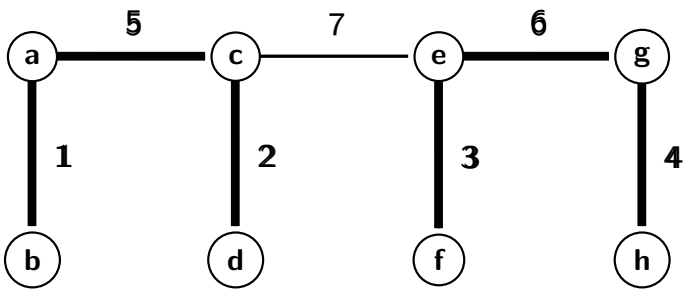

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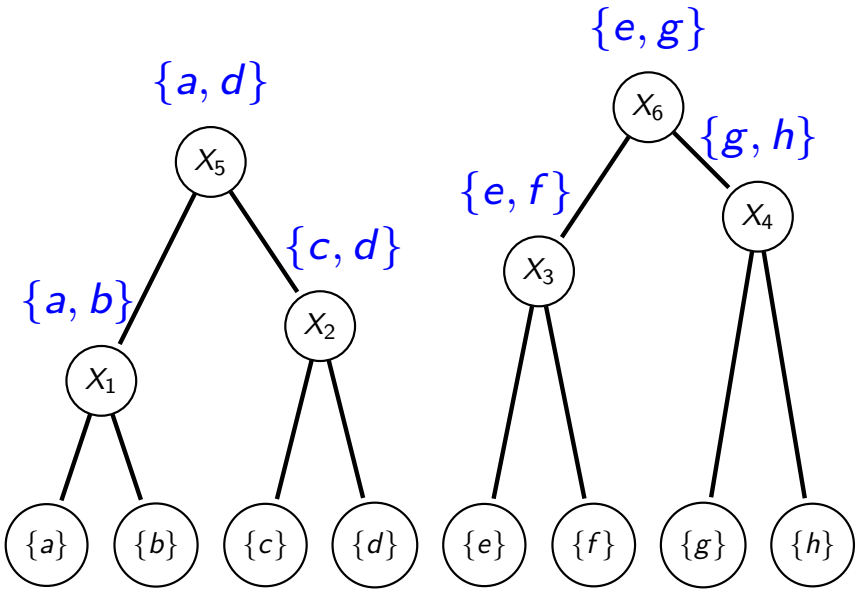

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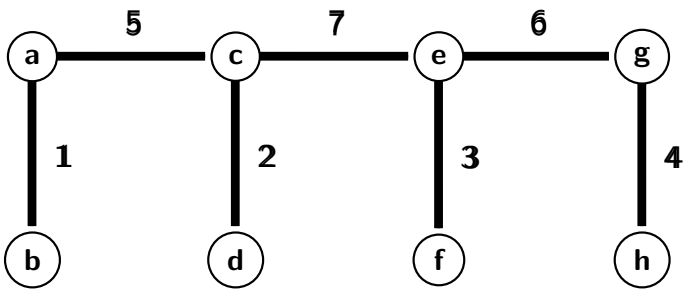

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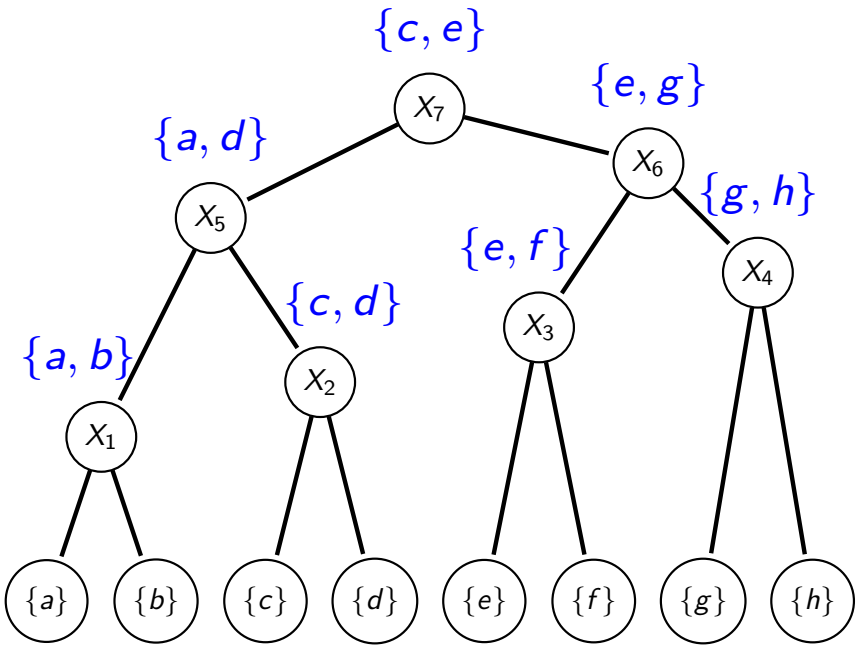

$\mathcal{B}$

Any non-leaf region R of $\mathcal{B}$ can be mapped to an edge u_R of G which is then called the building edge of R (shown in blue)

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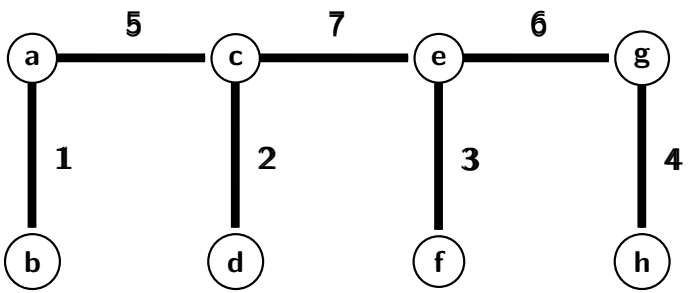
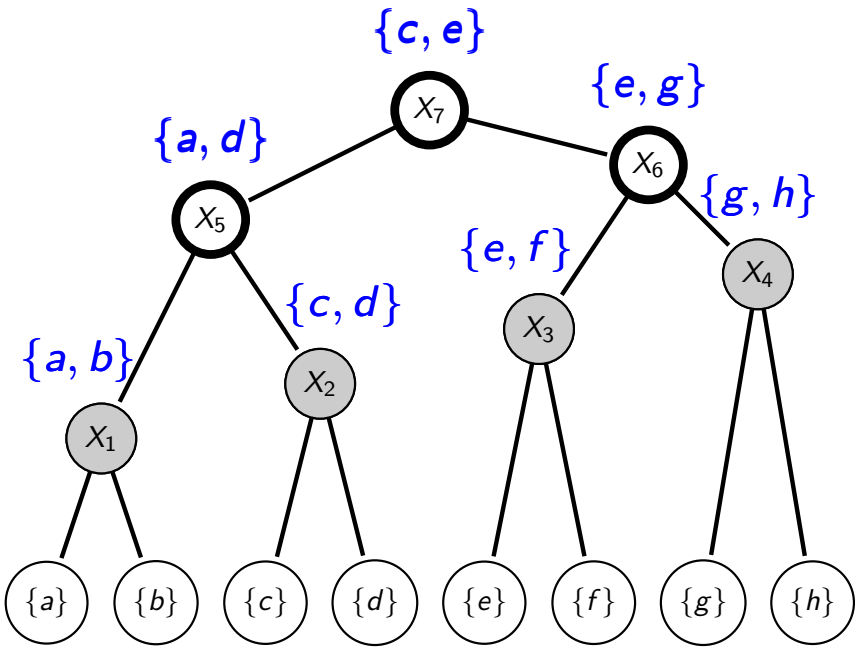


(G, w)



Any non-leaf region R of $\mathcal{B}$ can be mapped to an edge u_R of G which is then called the building edge of R (shown in blue)
Minima (in grey) and watershed-cut edges (in bold)

Watershed-cut ordering

In a hierarchical watershed, only the ordering of watershed-cut edges are considered

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3. $\Rightarrow$ watershed-cut ordering η_s
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6. verify if $\pi_s = s$

Definition

$$\eta_s(u) = \begin{cases} |\{v \in W \mid v \prec_s u\}| + 1 & u \in W \\ 0 & \text{otherwise} \end{cases}$$

, where W is the set of watershed edges of w and where $v \prec u$ if $s(u) < s(v)$ or if $s(u) = s(v)$ and $w(u) < w(v)$

Watershed-cut ordering

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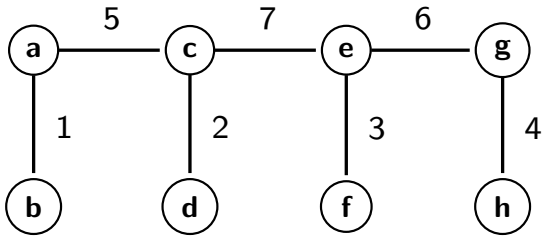

Weighted graph (G, w)

Definition

$$\eta_s(u) = \begin{cases} |\{v \in W \mid v \prec_s u\}| + 1 & u \in W \\ 0 & \text{otherwise} \end{cases}$$

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Watershed-cut ordering

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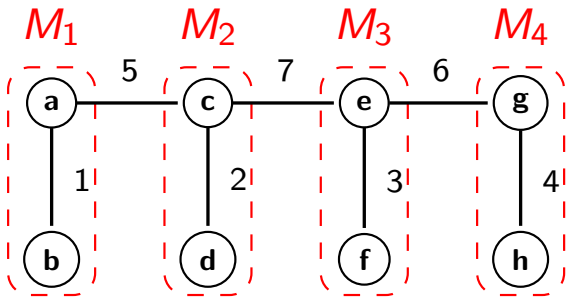

Minima of w (in red)

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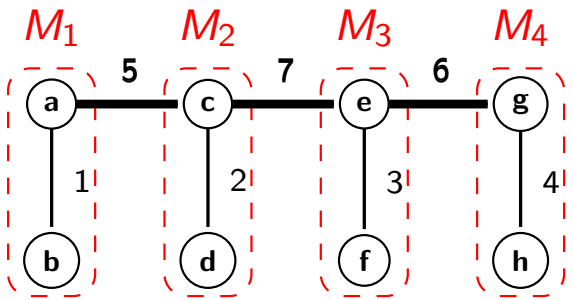

Minima of w (in red) and watershed-cut edges of w (in bold)

Definition

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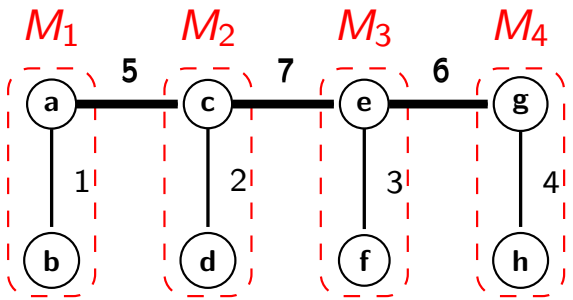
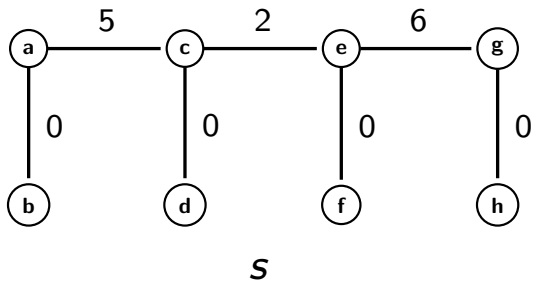


Minima of w (in red) and watershed-cut edges of w (in bold)



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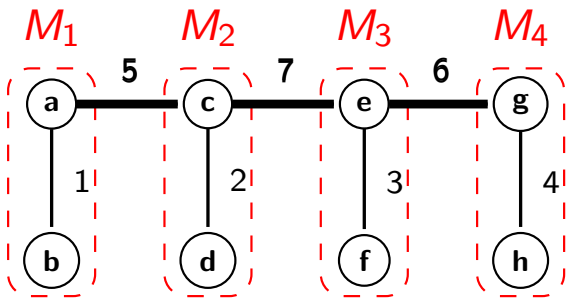
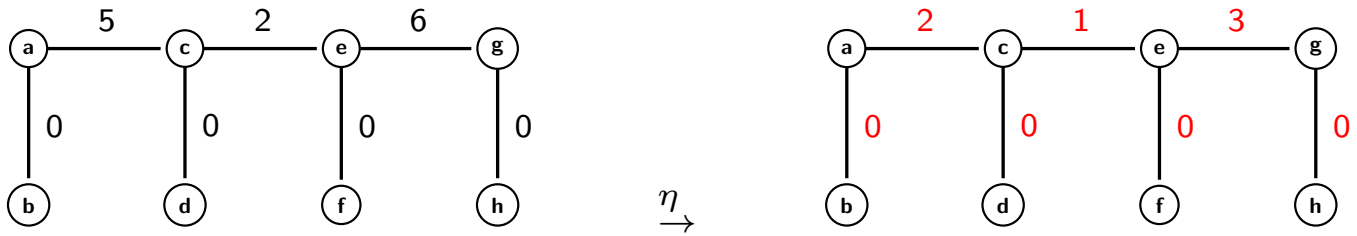


Minima of w (in red) and watershed-cut edges of w (in bold)



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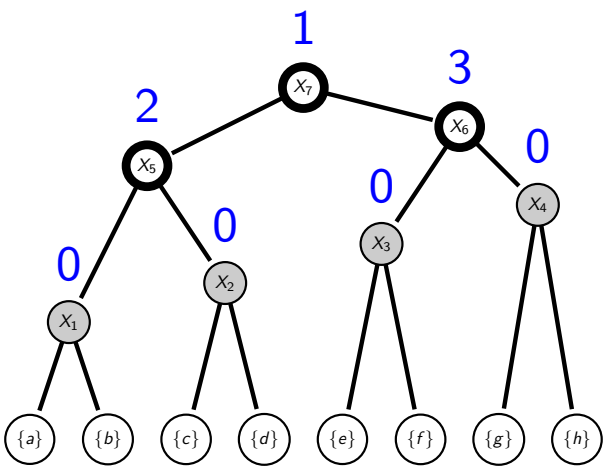
Extinction estimation



η_s

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Extinction estimation

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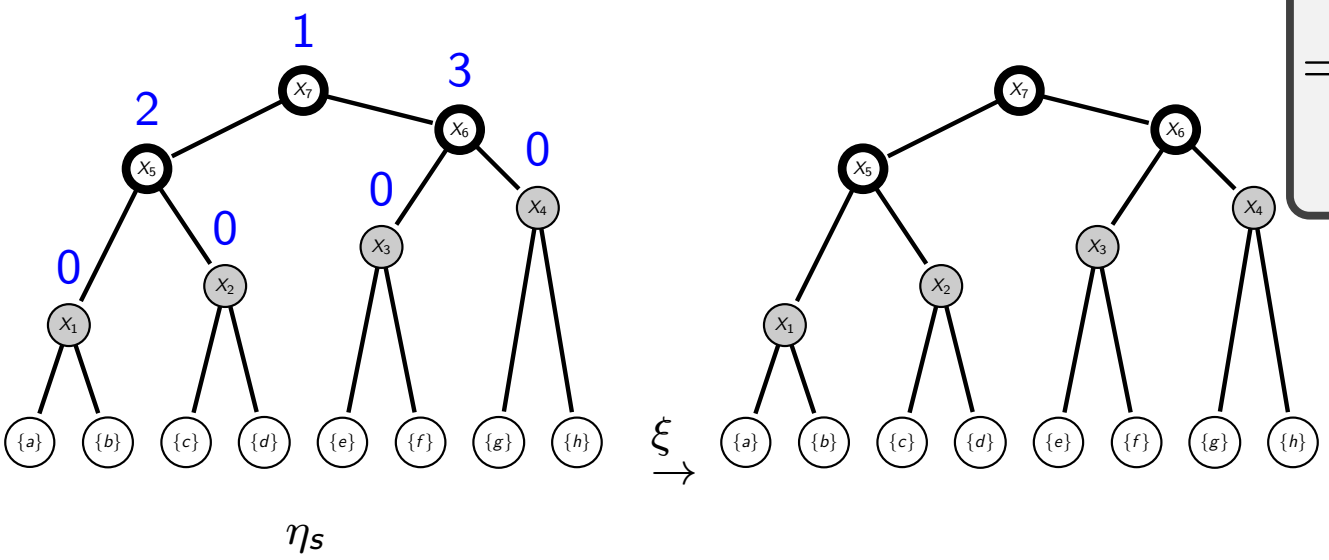

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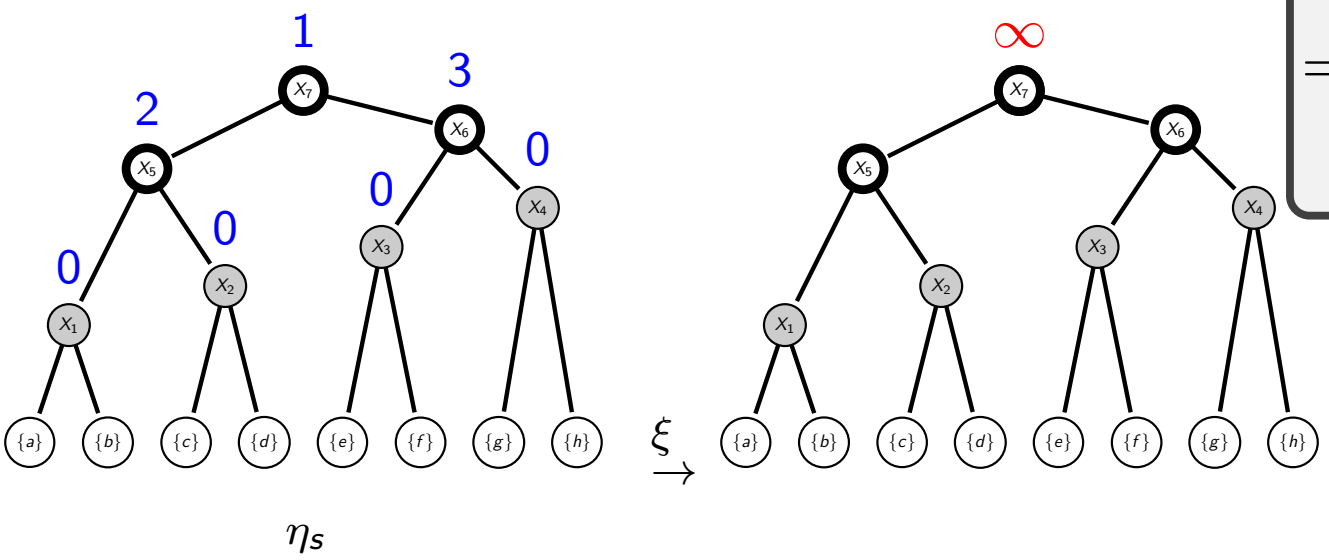

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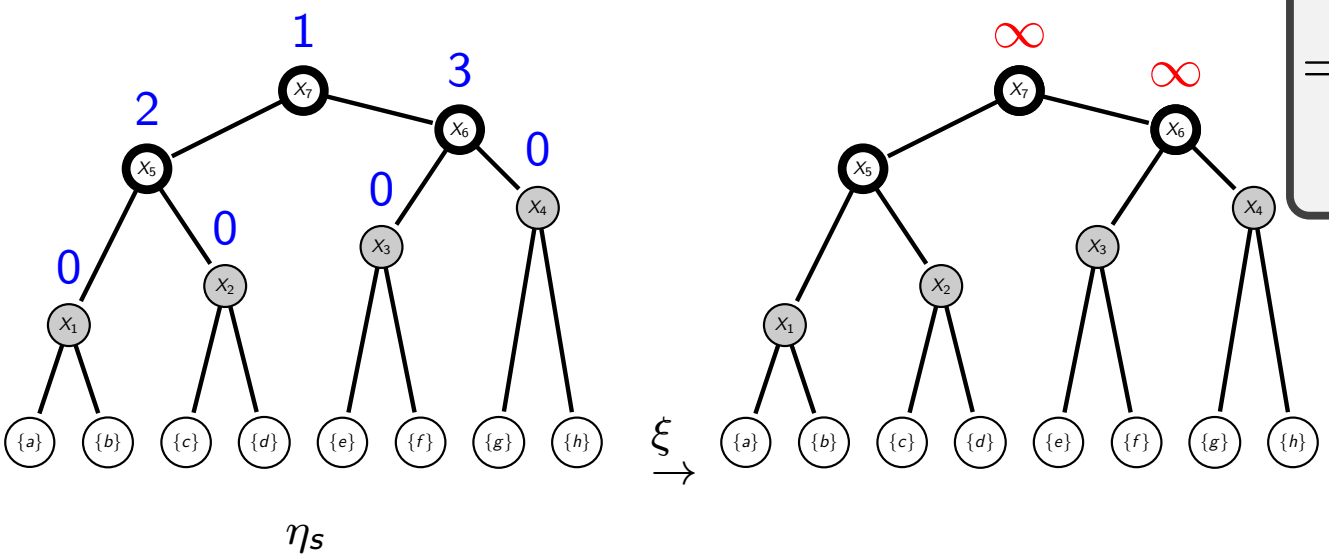

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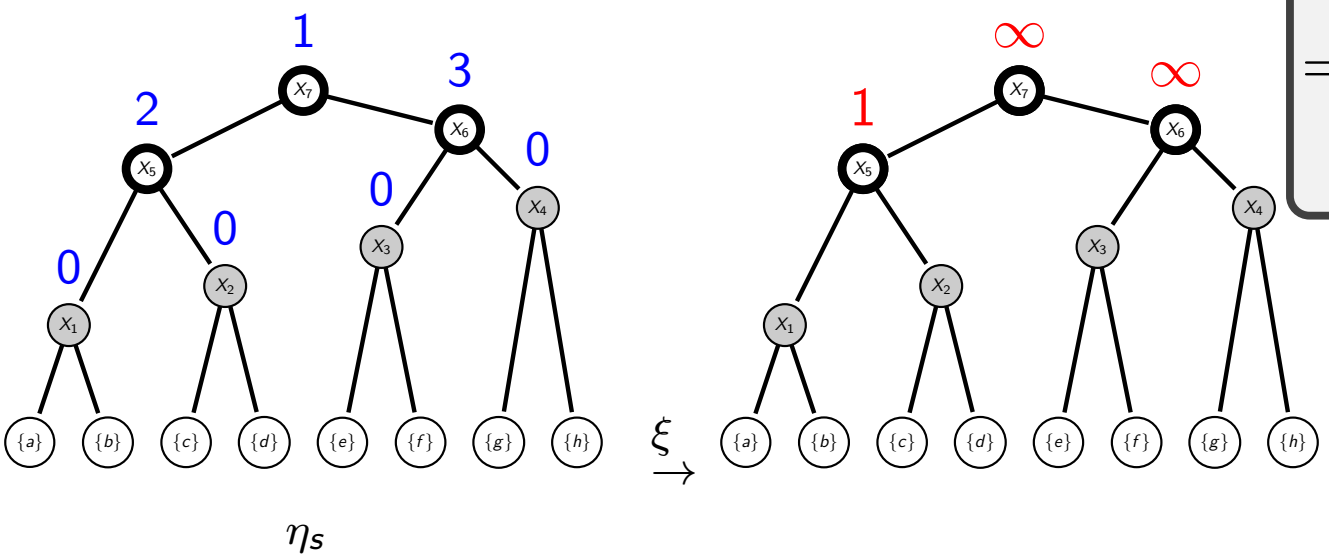

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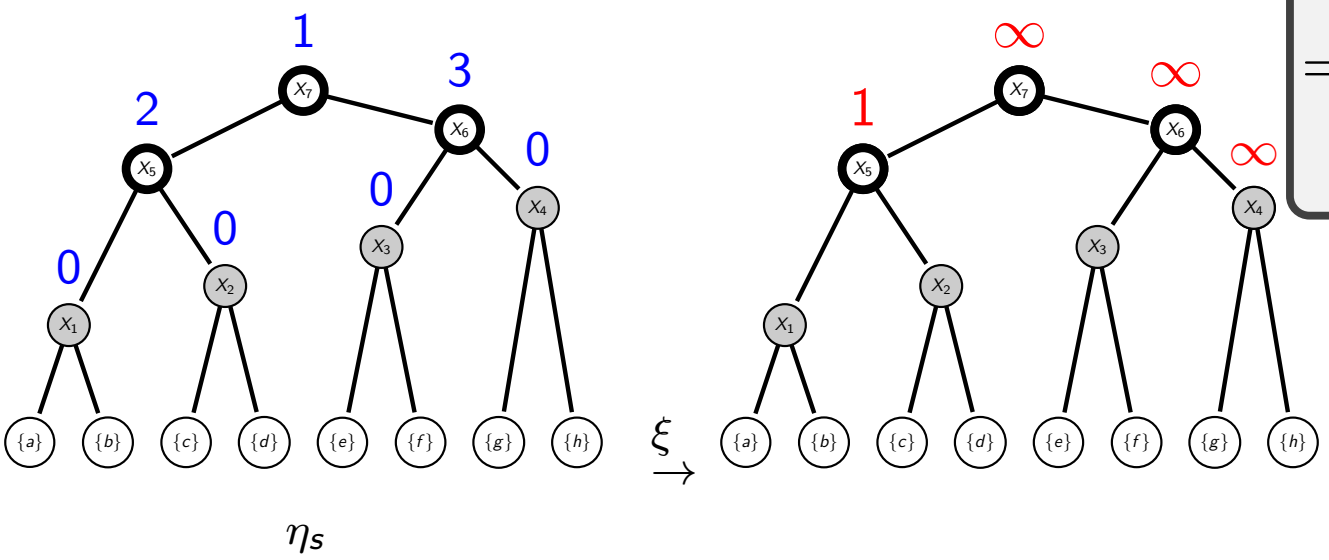

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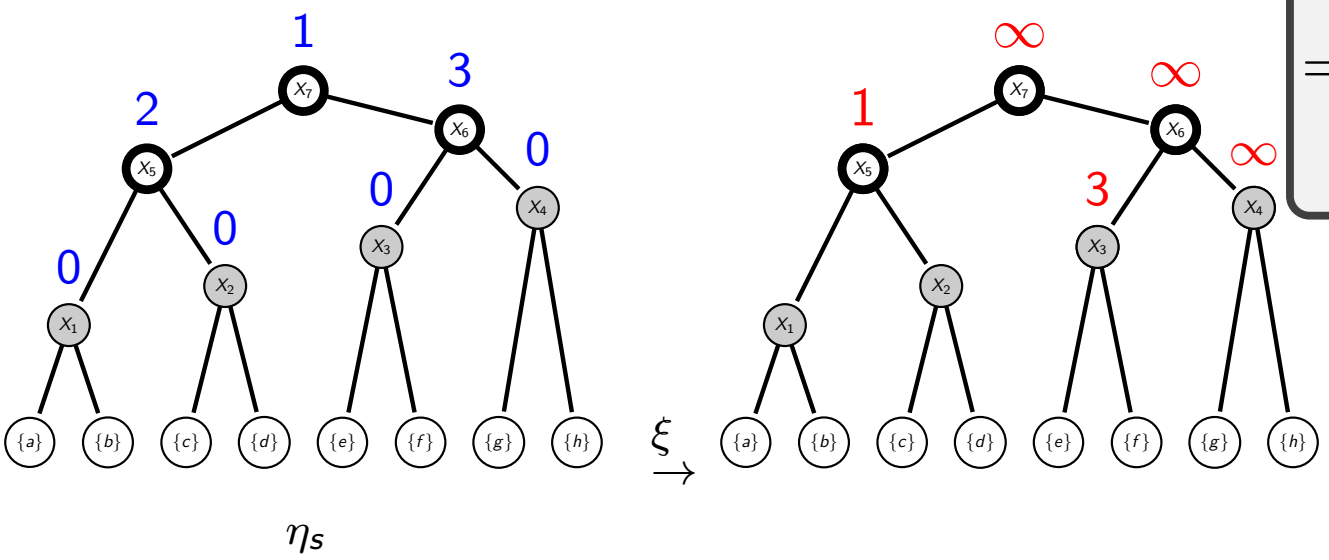

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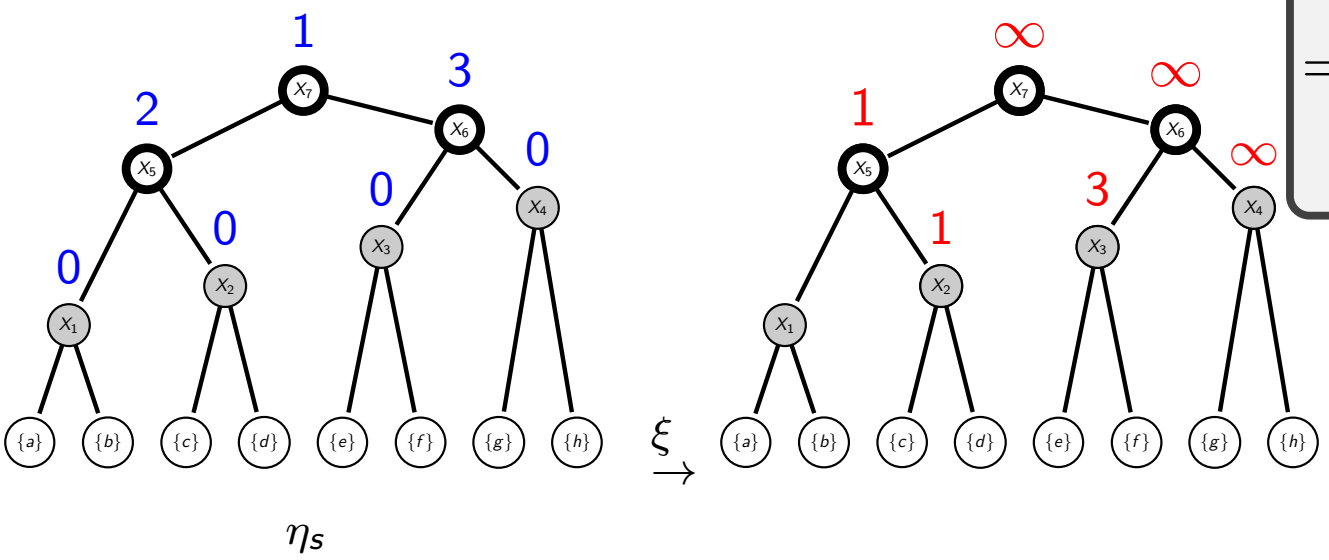

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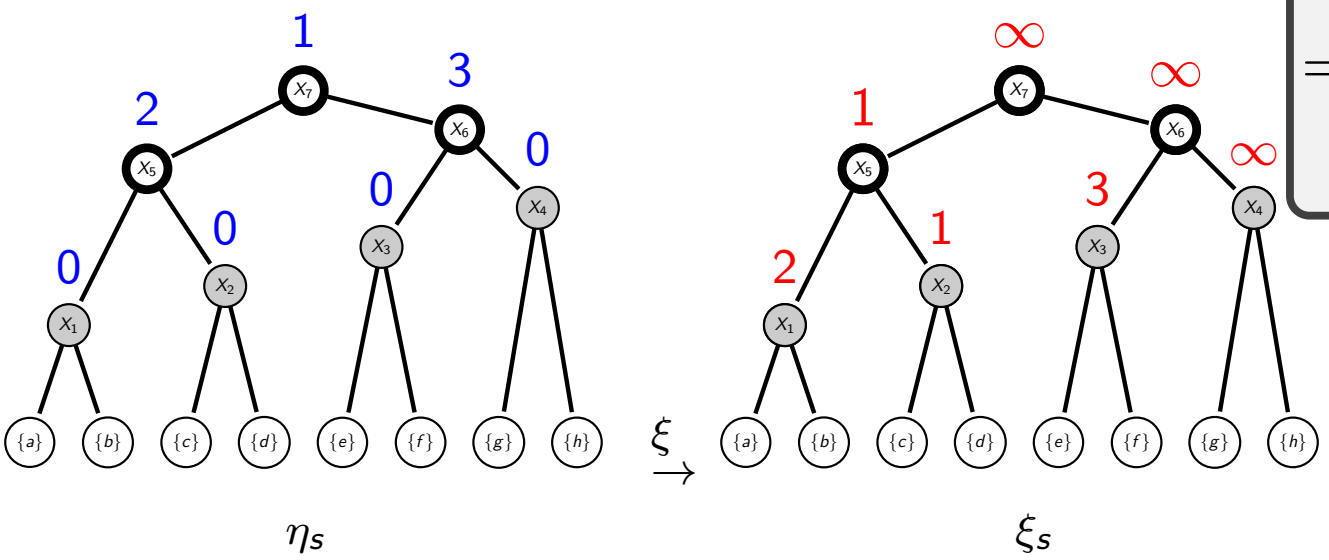

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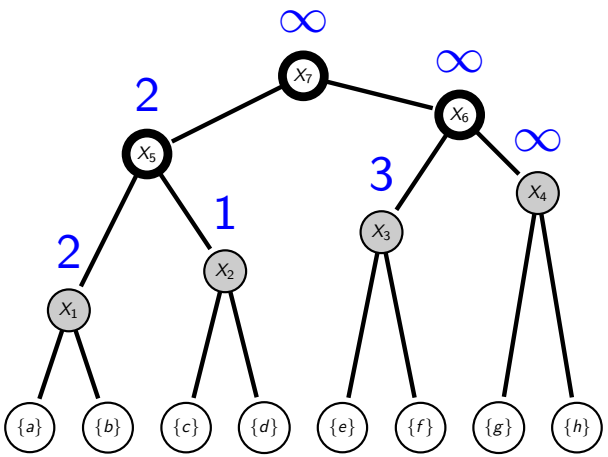
Persistence estimation



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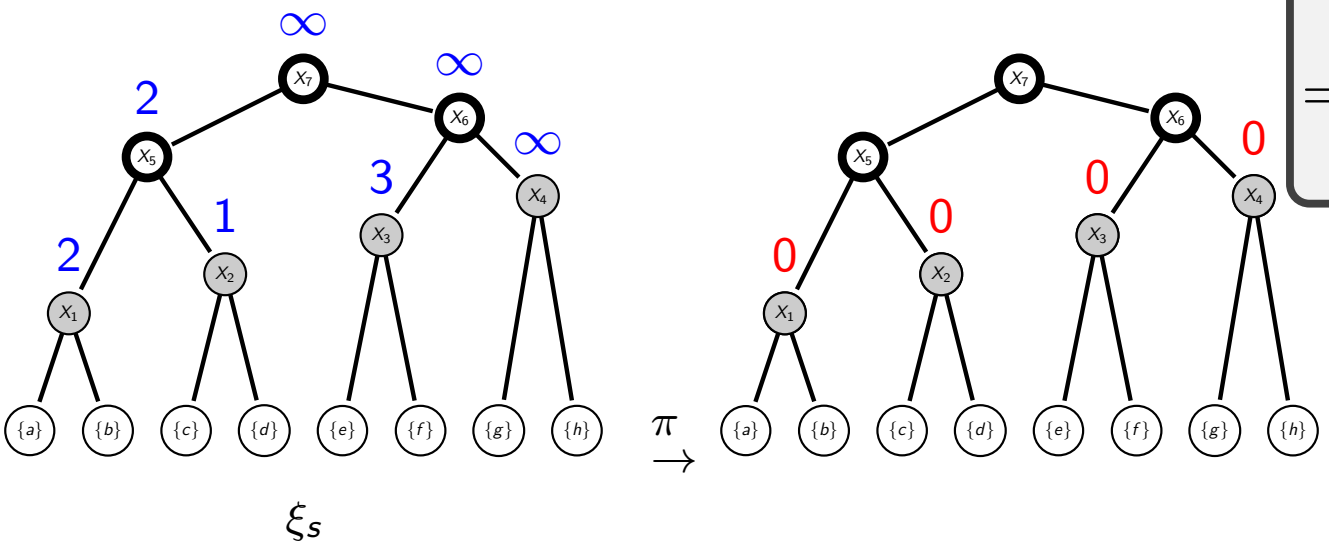

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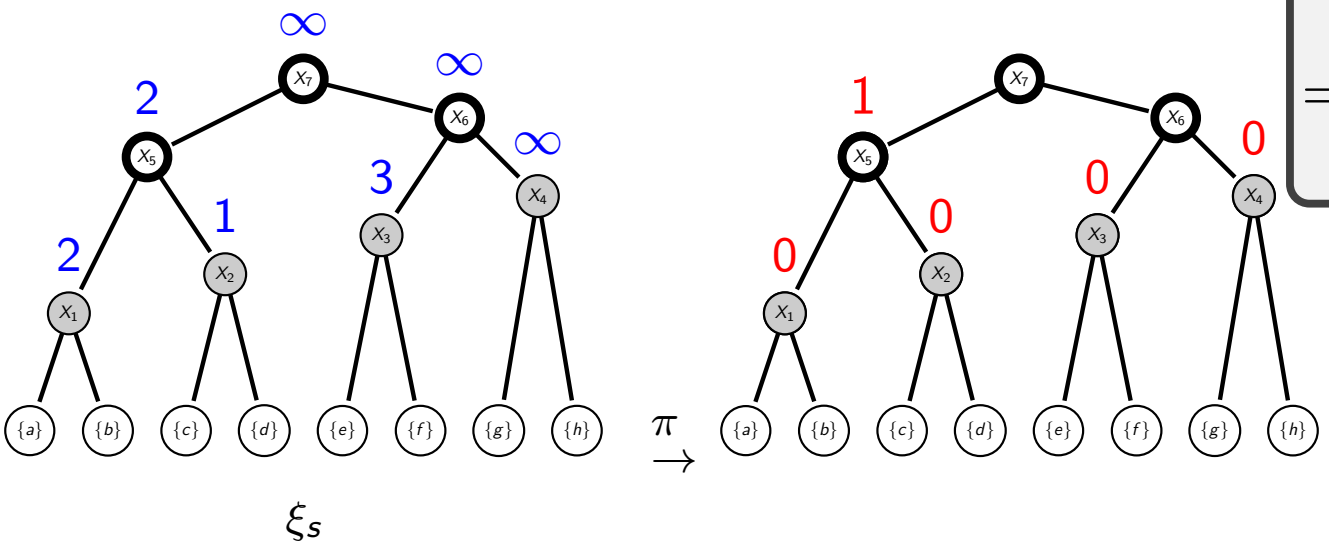

ξ_s

Definition

$$\pi_s(u) = \begin{cases} min\{\vee^{\xi_s}(v) \mid v \in E,\ parent(v) = u\} & \text{if } u \in W \\ 0 & \text{otherwise} \end{cases}$$

, where $\vee^{\xi_s}(u) = max\{\xi_s(v) \mid v \in E, R_v \subseteq R_u\}$ and where W is the set of watershed edges of w.

Persistence estimation

Recognition algorithm:
1. binary partition hierarchy $\mathcal{B}$
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3. watershed-cut ordering η_s
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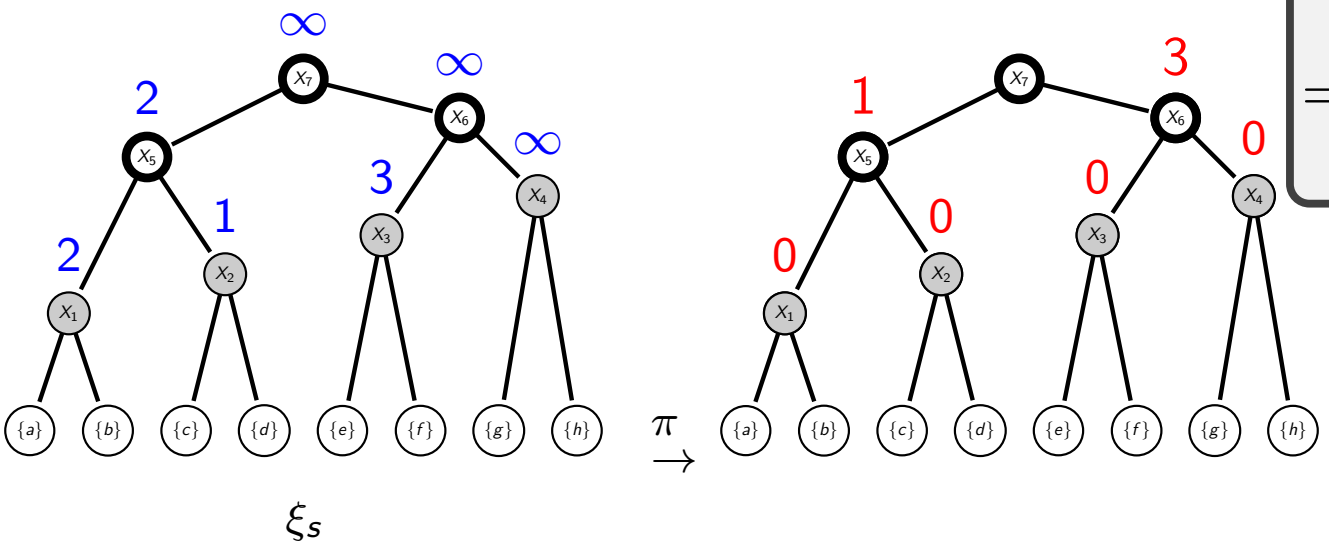

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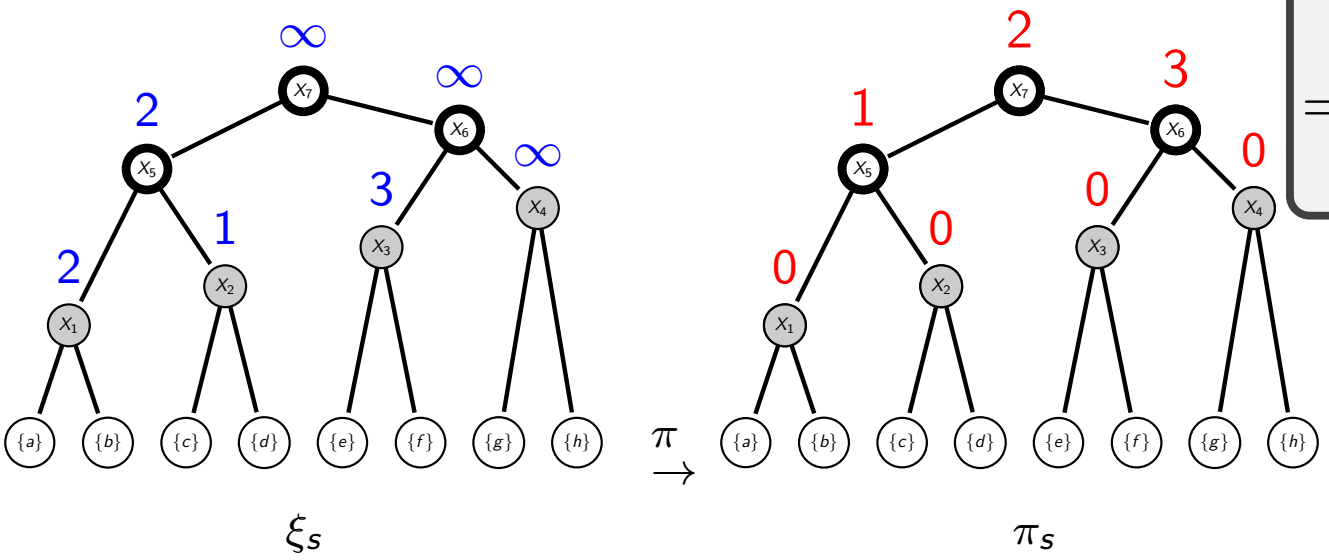

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Let $\mathcal{H}$ be a hierarchy and let s be the saliency map of $\mathcal{H}$.

Definition

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Consequence: The proposed algorithm recognizes hierarchical watershed

Outlines

Conclusion and perspectives

Summary

- *Introduction of watersheding transform*
- *Problem of recognition of hierarchical watersheds solved by the watersheding transform*
- *Hierarchical watersheds are the fixed points of the watersheding transform*
- *Quasi-linear algorithm to the watersheding proposed*

Perspectives

- *Answer the question: does the combination of hierarchical watersheds always result in a hierarchical watershed?*
- *Extension to arbitrary weighted graphs*
- *Frequency study of hierarchical watersheds*
- *Practical interest of the watersheding transform*