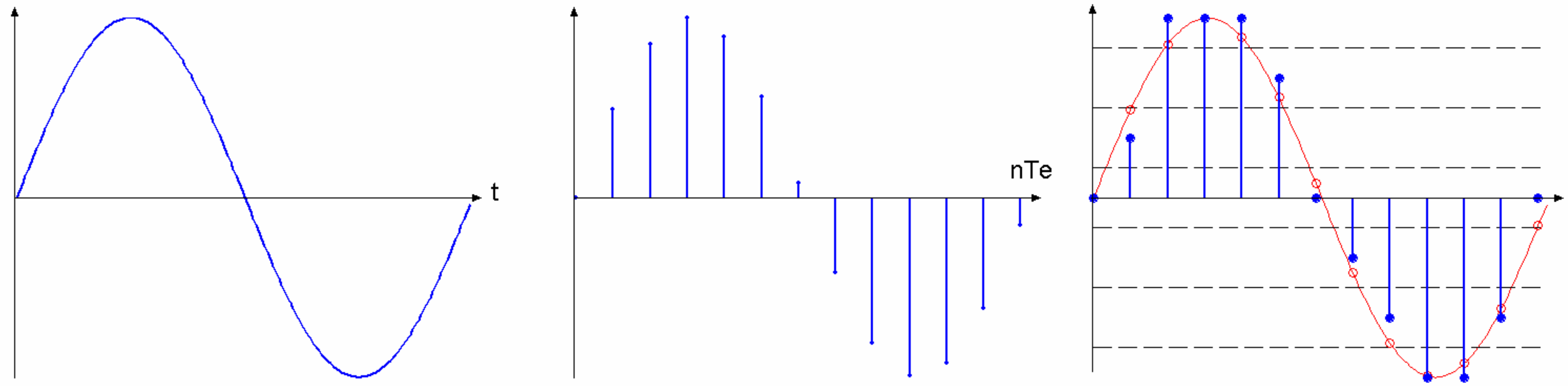
Echantillonnage et Quantification

De l’analogique au numérique

Echantillonnage
discrètisation de
l’axe des abscisses

Quantification
discrètisation de
l’axe des ordonnées



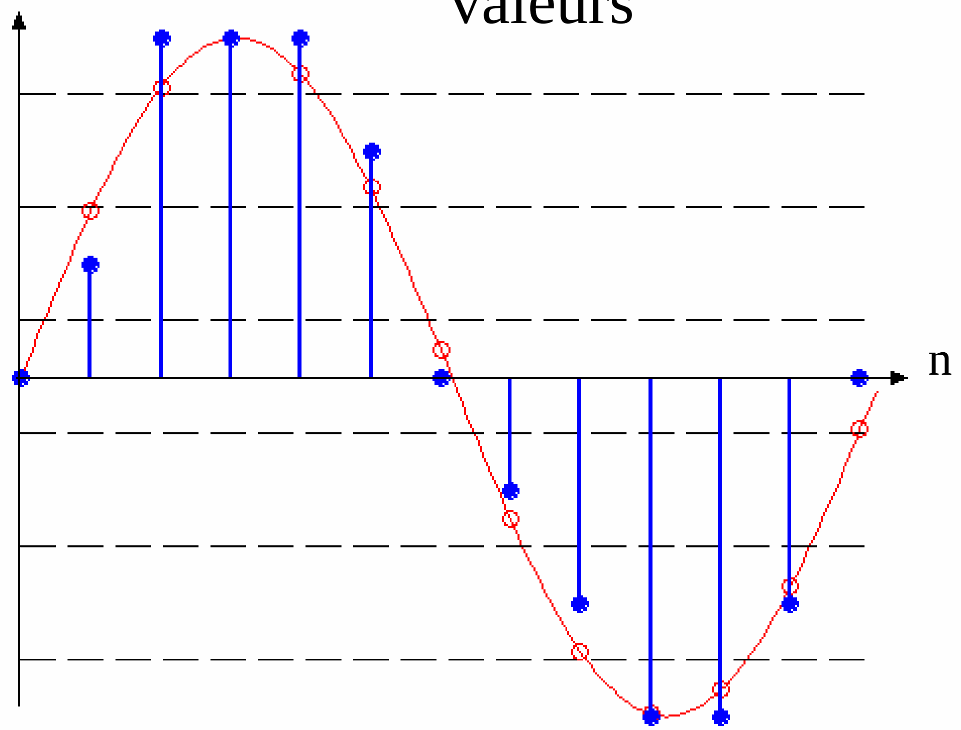
réversible

irréversible

Du numérique au traitement

Codage
Représentation des valeurs

011
010
001
000/100
101
110
111



000
001
011
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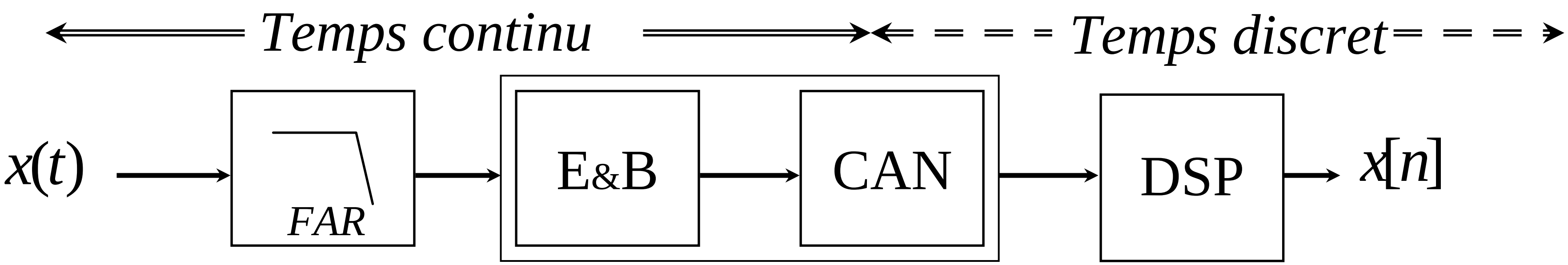
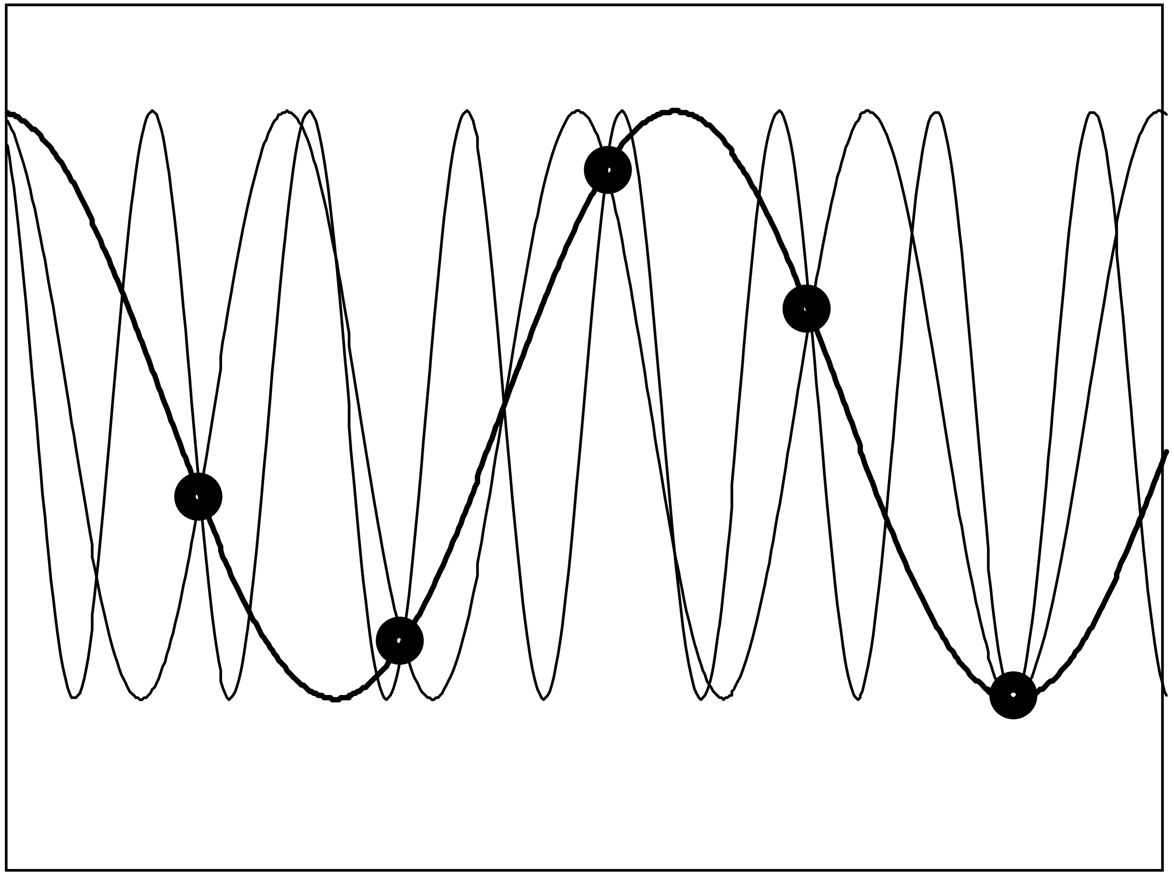
011
010
000
101
110
…

réversible

Chaîne de traitement



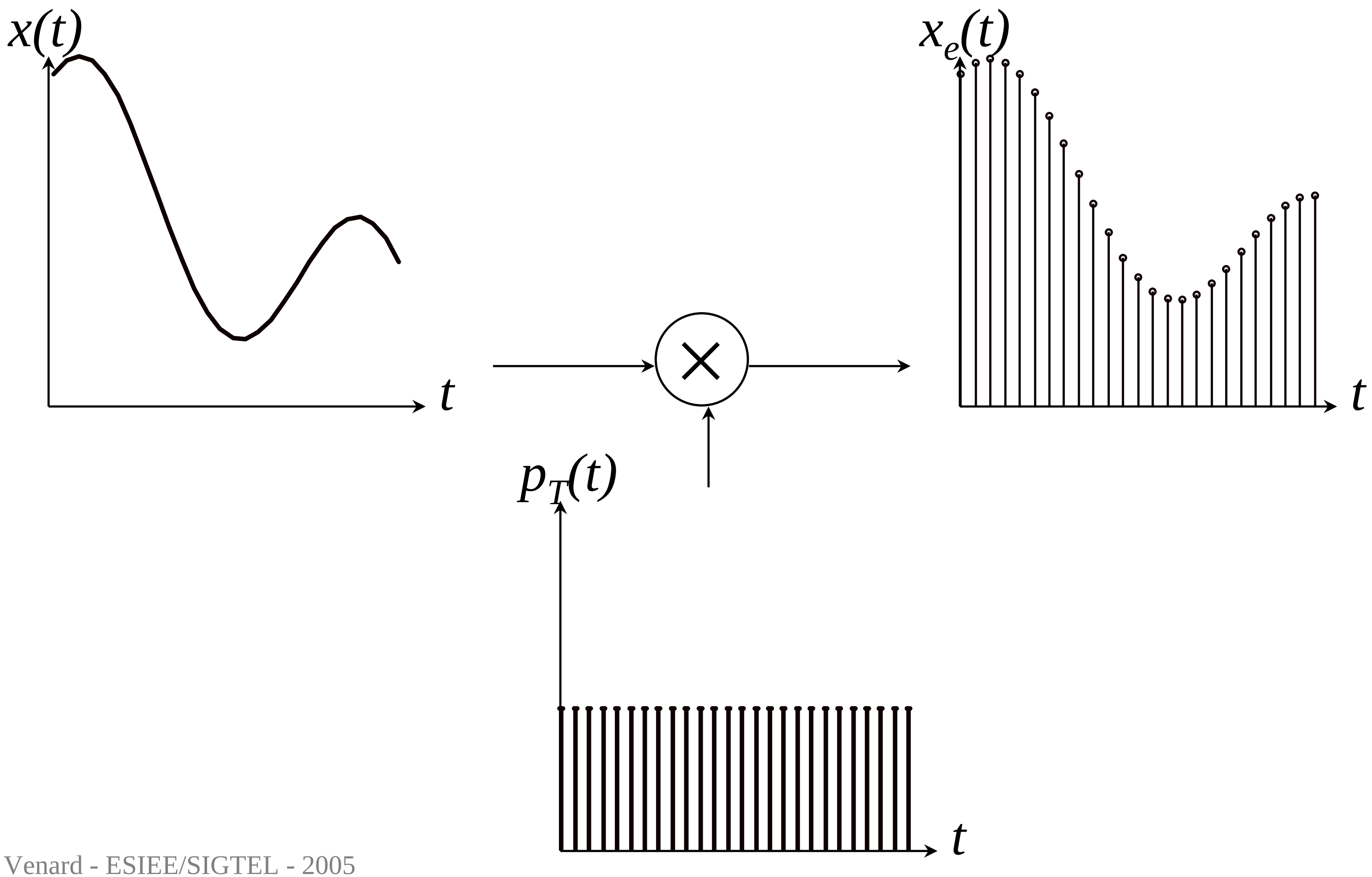
Echantillonnage



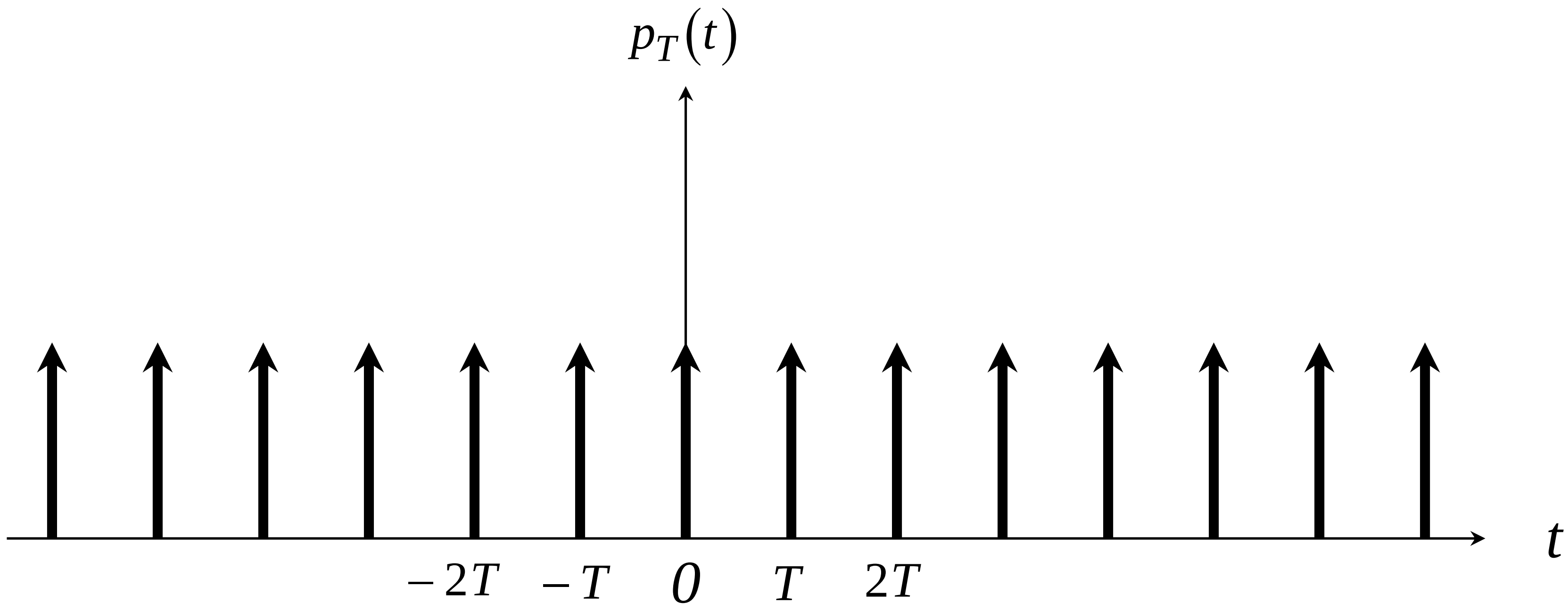
f_1=2KHz
f_2=8KHz
f_3=12KHz

Fe=10KHz

Modèle de l'échantillonnage

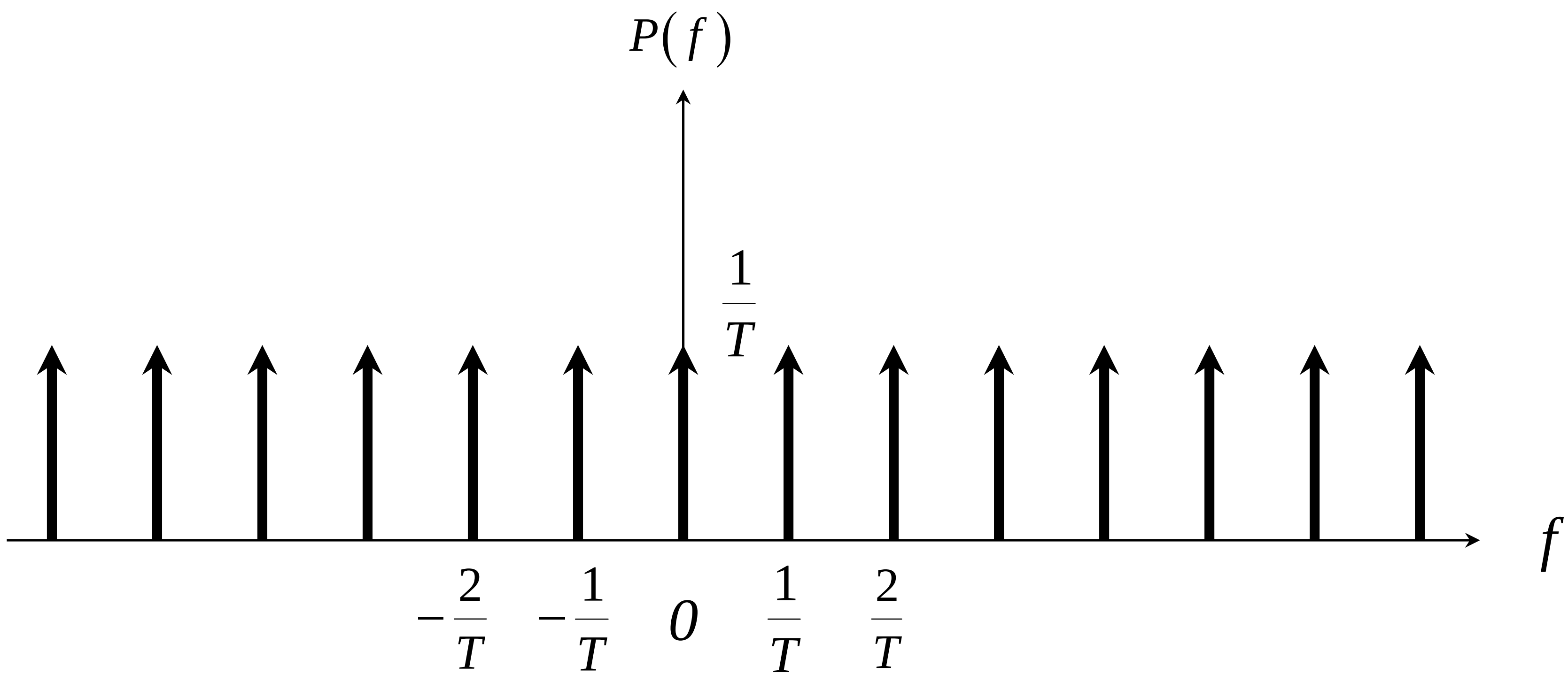


Aparté 1
(Train d'impulsion)



$$p_T(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

TF d’un train d’impulsion



$$P(f) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} \delta\left(f - k\frac{1}{T}\right)$$

Aparté 1
(Théorème du fenêtrage)

$$x(t).y(t) \underset{TF^{-1}}{\overset{TF}{\rightleftarrows}} \int_{-\infty}^{+\infty} X(\upsilon)Y(f-\upsilon)d\upsilon$$

…(suite)

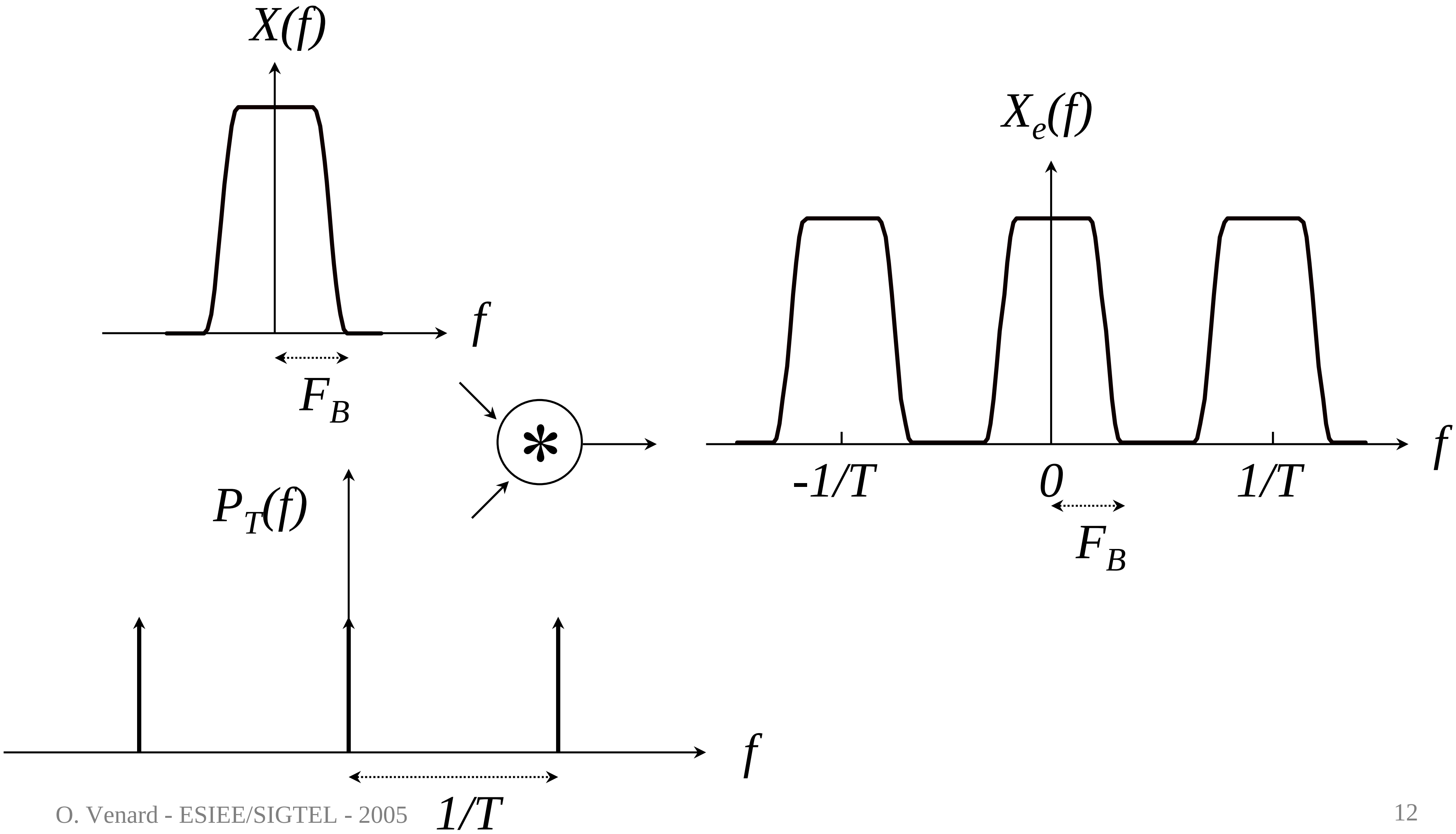
$$x_e(t) = x(t) \cdot p_T(t) = x(t) \cdot \sum_{n=-\infty}^{+\infty} \delta\left(t - nT\right)$$

$$\Downarrow \; TF$$

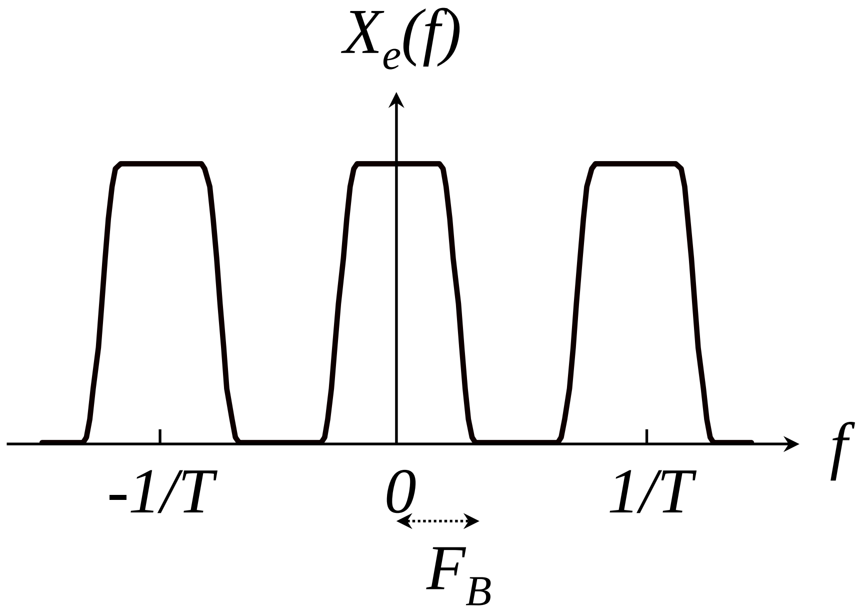
$$X_e(f) = X(f) * P_T(f) = X(f) * \frac{1}{T} \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{T}\right)$$

$$\boxed{X_e(f) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X\left(f - \frac{k}{T}\right)}$$

…(suite)

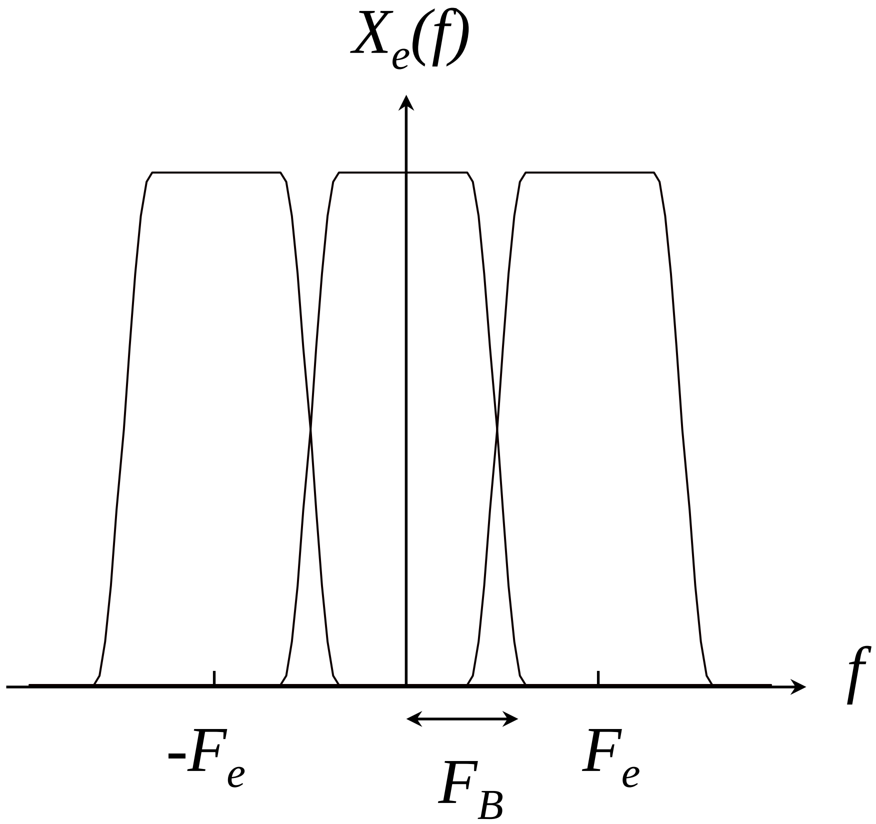


Théorème d'échantillonnage



$$F_e = \frac{1}{T} \geq 2F_B$$

Repliement de spectre



$$F_B > \frac{F_e}{2}$$

Original (BW 4Khz)

Echantillonné (2Khz)

Original filtré (1Khz)

Filtré échantillonné (2Khz)

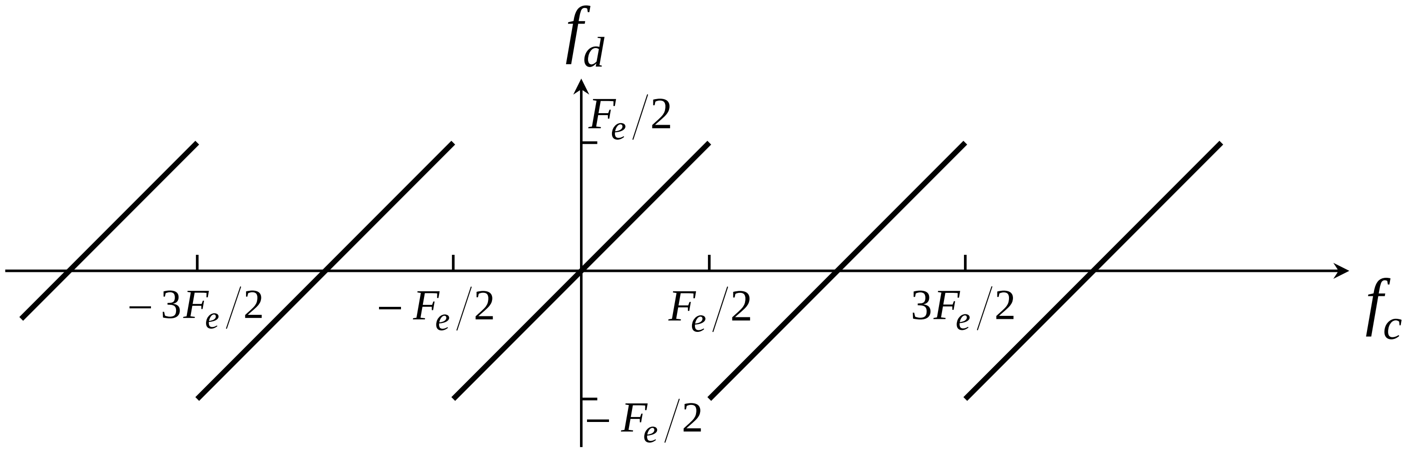
…(suite)

Soit le signal continu :

$$x(t) = A\cos\left(2\pi f_k t + \theta\right) \text{ avec } -\infty < f_k < +\infty,$$

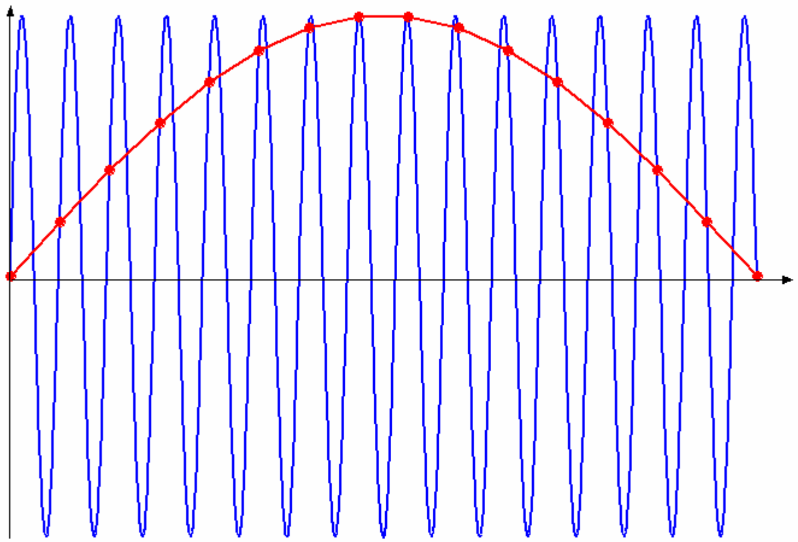
$$\text{si } f_k = f_0 + kF_e \text{ avec } \left|f_0\right| < \frac{F_e}{2} \text{ et } k = 0,1,2,\cdots$$

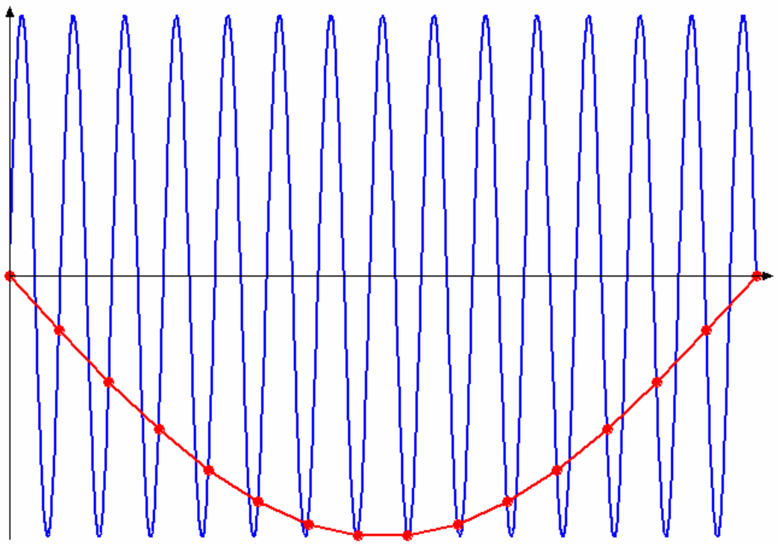
Le signal échantillonné sera : $x(t) = A\cos\left(2\pi f_0 n T_e + \theta\right).$



…(suite)

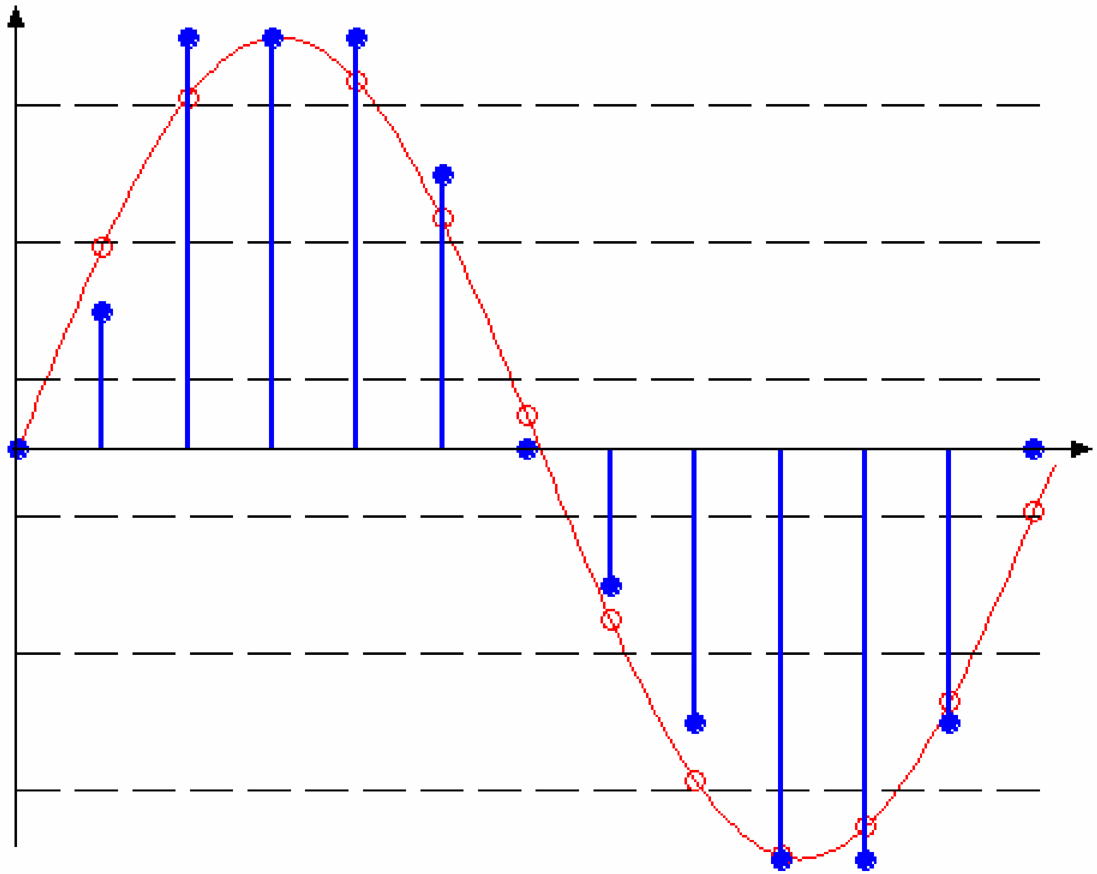
$$F_e = 600Hz$$
$$f_k = 620Hz$$
$$f_k = 20Hz + F_e$$



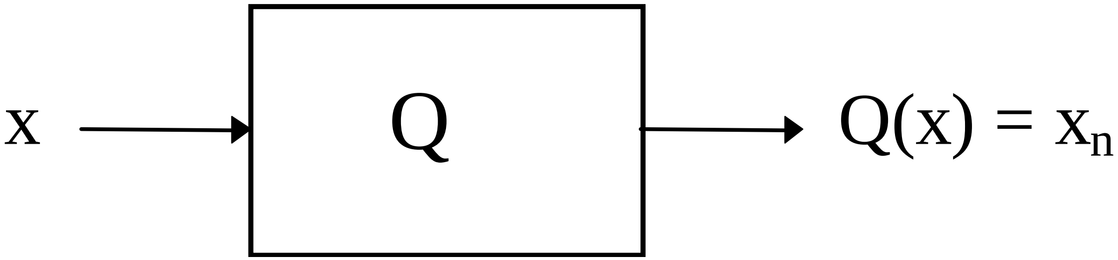


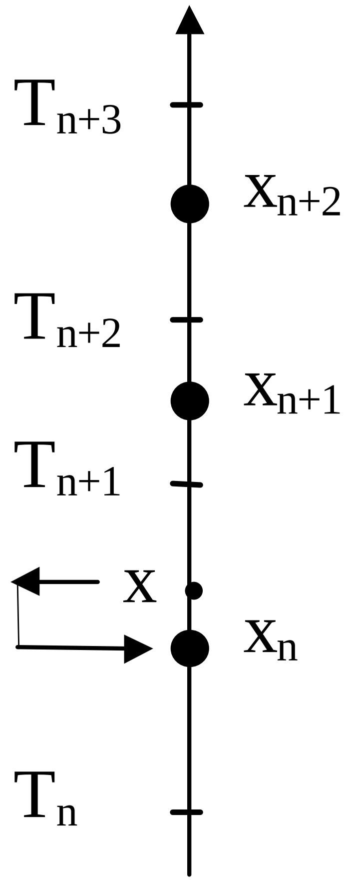
$$F_e = 600Hz$$
$$f_k = 580Hz$$
$$f_k = F_e - 20Hz$$

Quantification



…(suite)





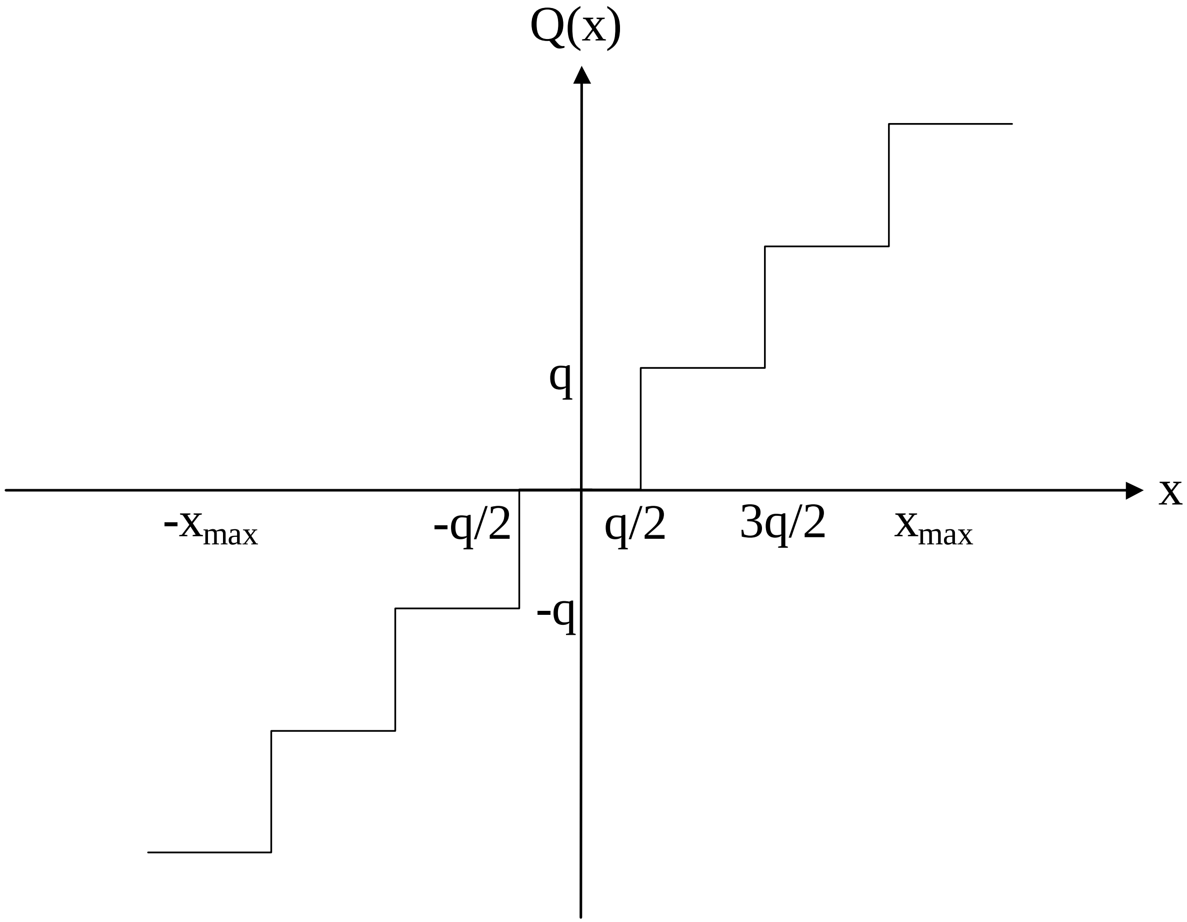
$$\forall x \in [T_n, T_{n+1}]$$

$$Q(x) = x_n$$

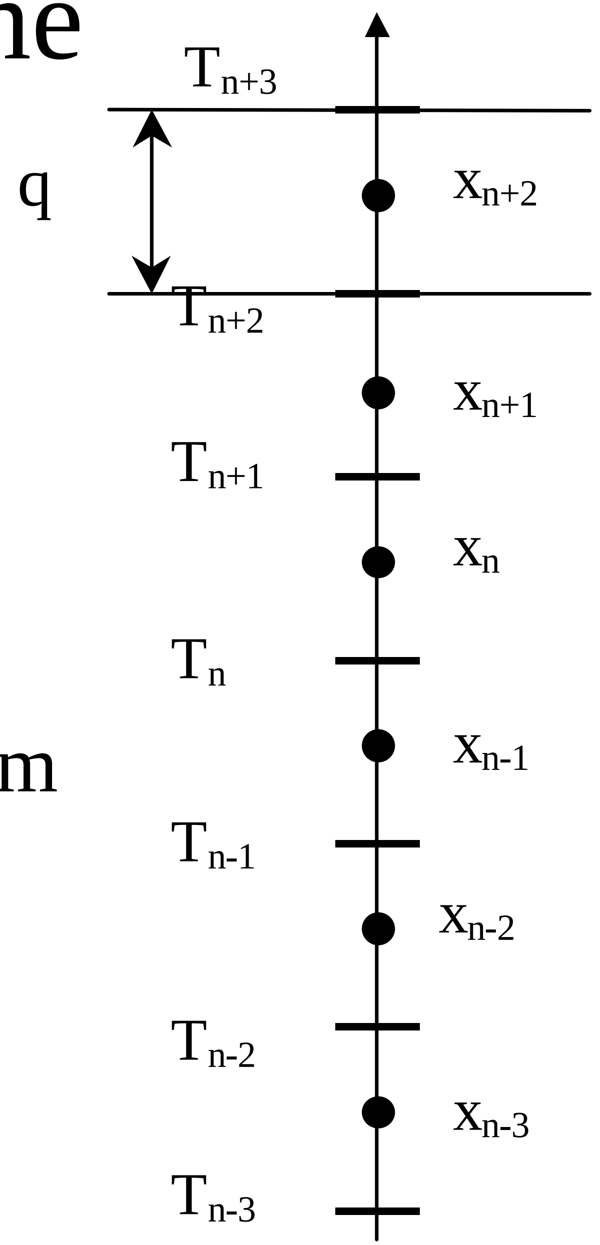
T_n = Seuil de Quantification

x_n = Valeur de Quantification

…(suite)



Quantification uniforme

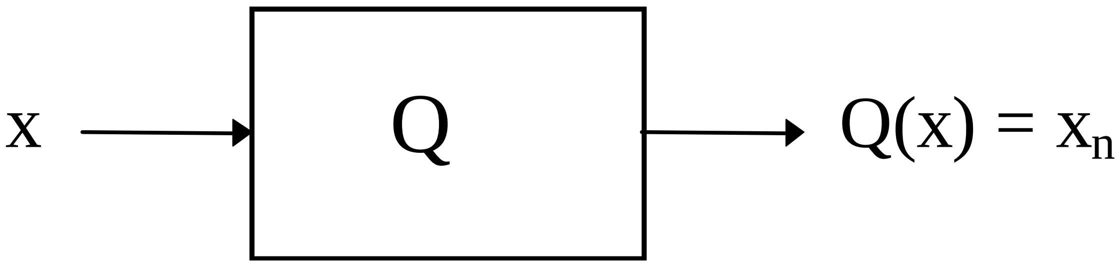


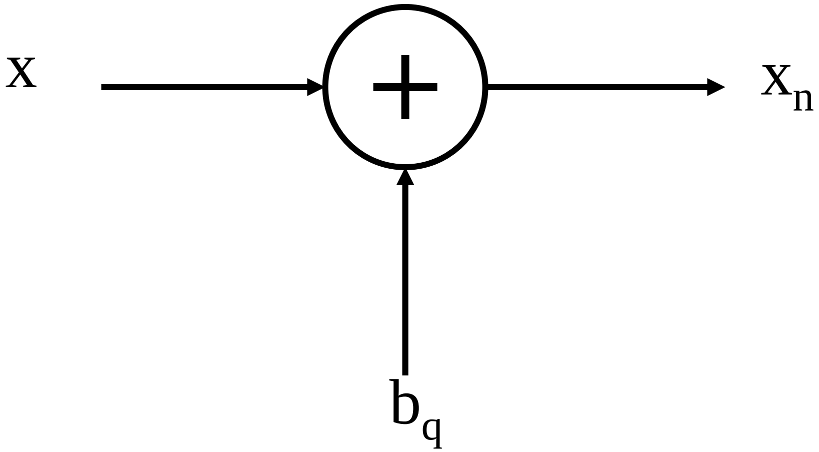
$$\forall n \quad \left|T_{n+1} - T_n\right| = q$$

$$\forall n \quad \left|x_{n+1} - x_n\right| = q$$

$q =$ pas de quantification = quantum

Dégradation





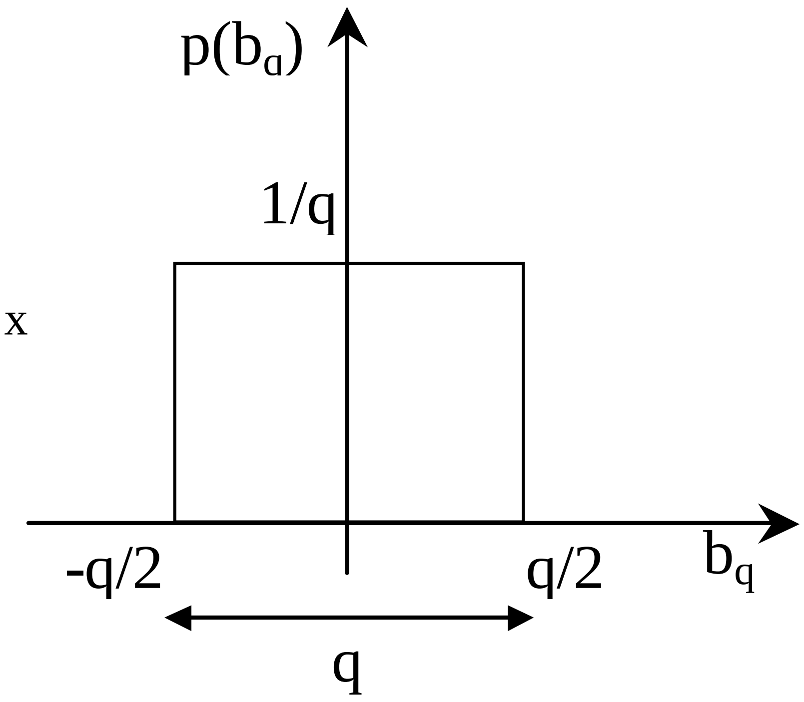
Modèle probabiliste

$$b_q = \hat{x} - x \quad \text{(arrondi)}$$

$$\left|b_q\right| \leq \frac{q}{2} \qquad \text{pour } x \leq x_{max}$$

$$E(b_q) = 0$$

$$E\left(b_q^2\right) = \sigma_b^2 = \frac{q^2}{12}$$



Evaluation de la dégradation

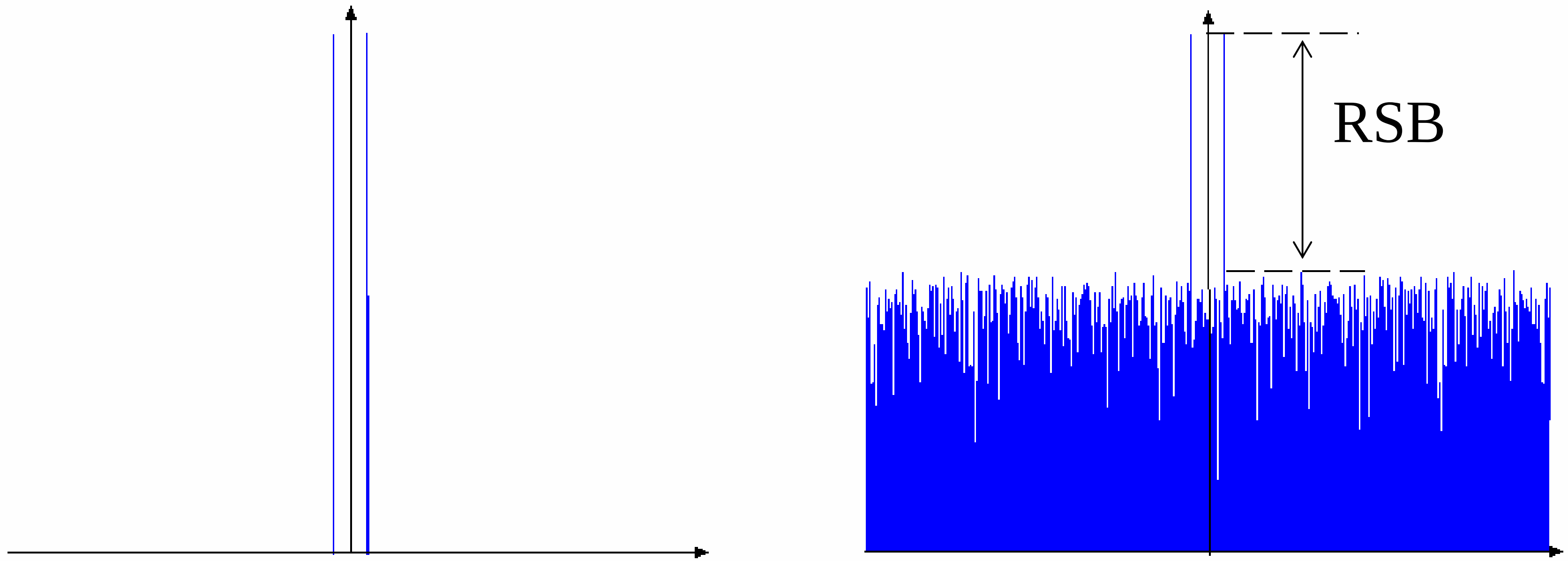
$$q = \frac{2x_{\max}}{2^N - 1}$$

$$RSB_q = 10\log\left(\frac{\sigma_x^2}{\sigma_b^2}\right) = 10\log\left(\frac{\sigma_x^2}{x_{\max}^2}\right) + 4.77 + 6.02N \text{ dB}$$

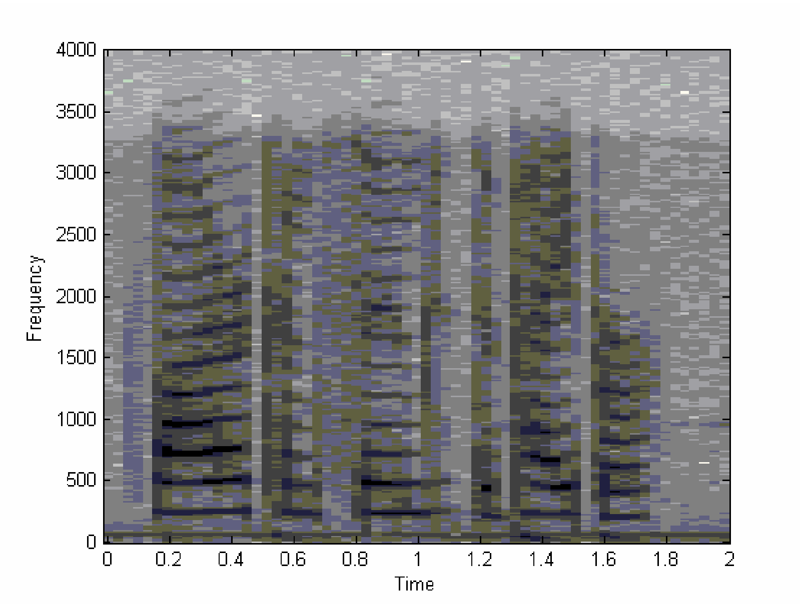
Pour un sinus d'amplitude 1:

$$RSB_q = 1.76 + 6.02N \text{ dB}$$

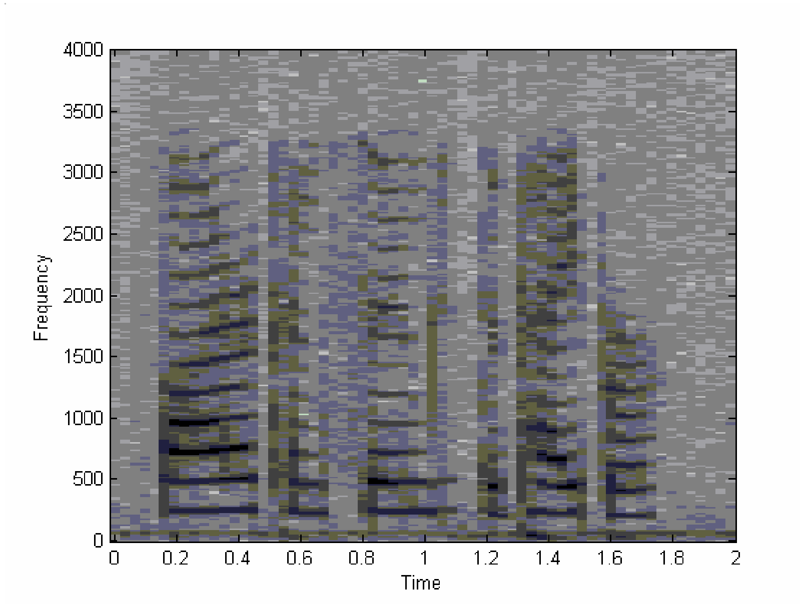
Nombre de bits effectifs ENOB



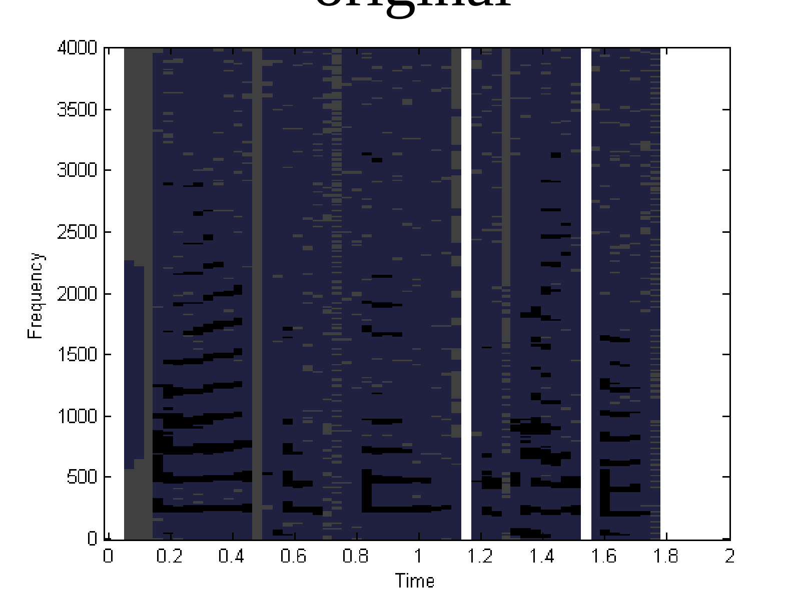
$$N_{ENOB} = \frac{RSB - 1.76}{6.02}$$



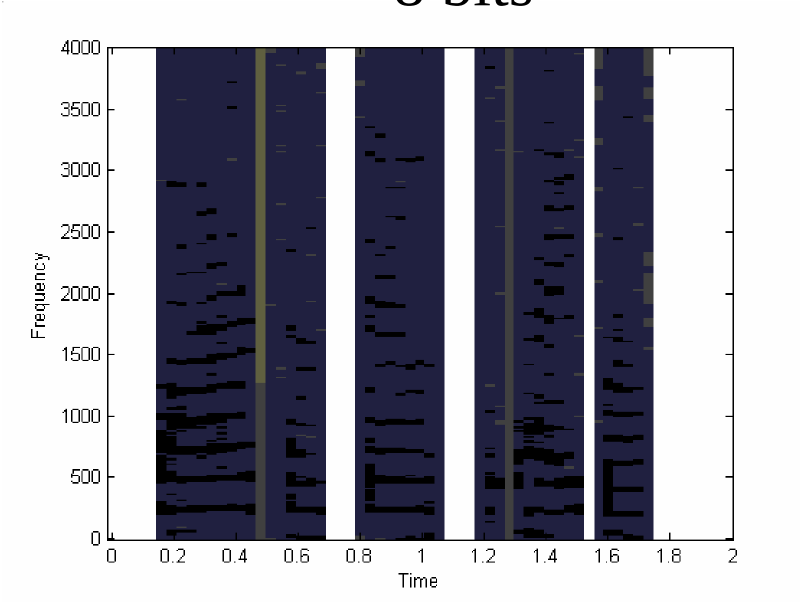
original



8 bits

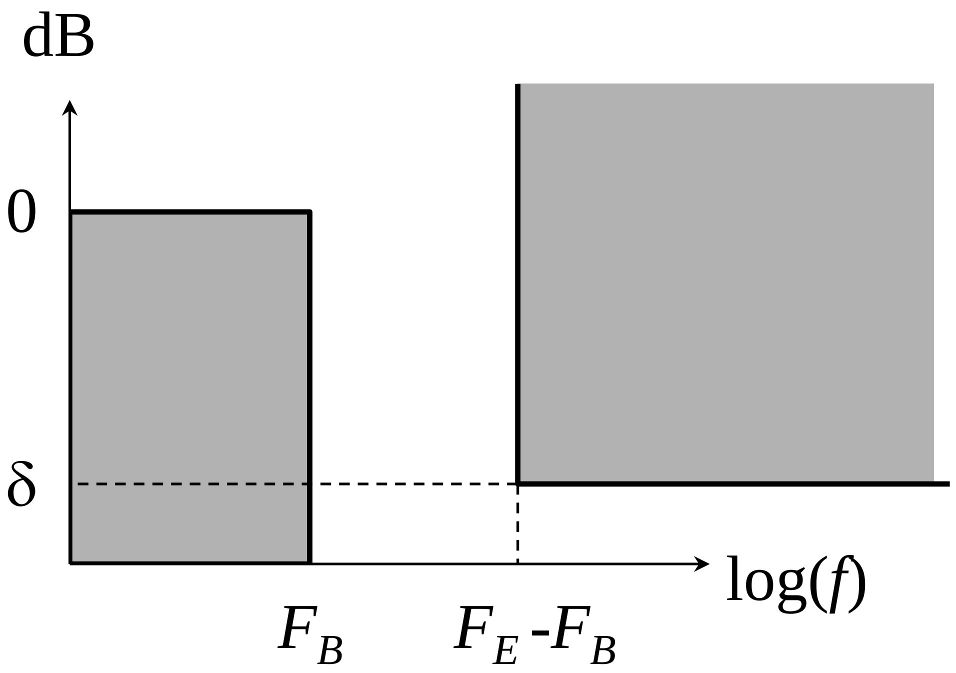


6 bits



4 bits

Filtre anti-repliement

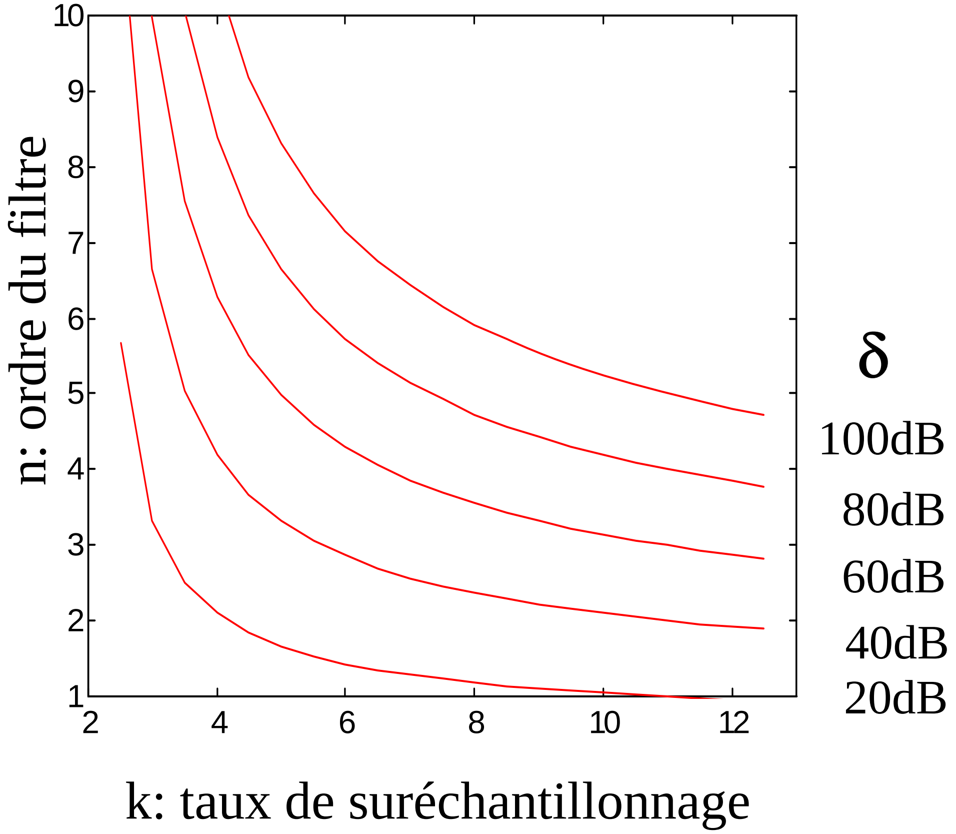


$$n = \frac{\delta}{\log(k-1)} \frac{\log(10)}{20}$$

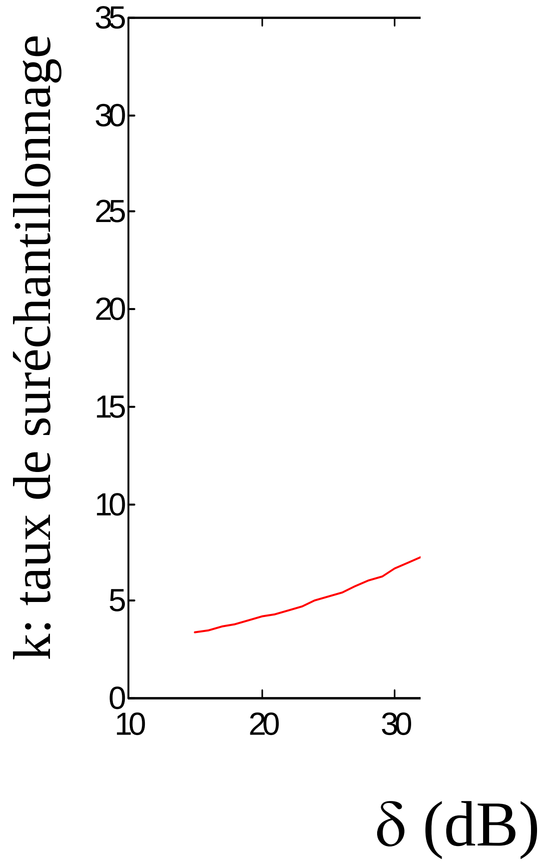
n: ordre du filtre AR
k: taux de suréchantillonnage

…(suite)

Atténuation

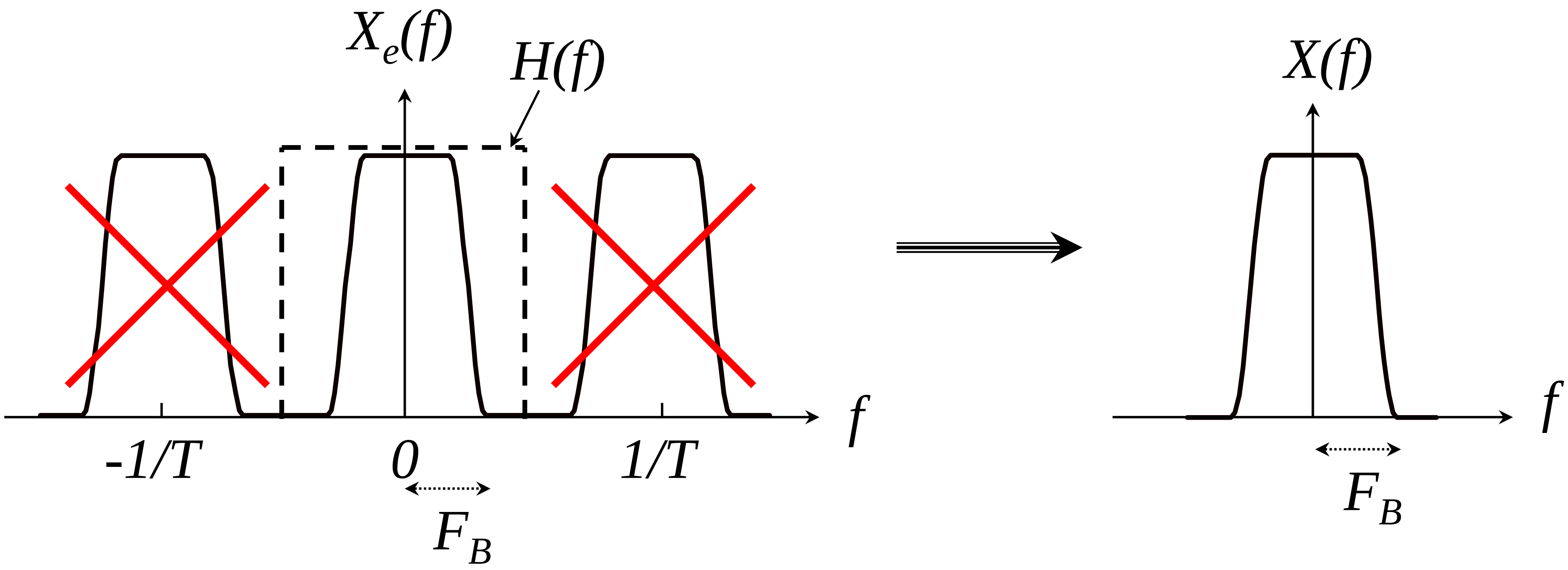


Suréchantillonnage
(filtre d'ordre 2)



Reconstruction

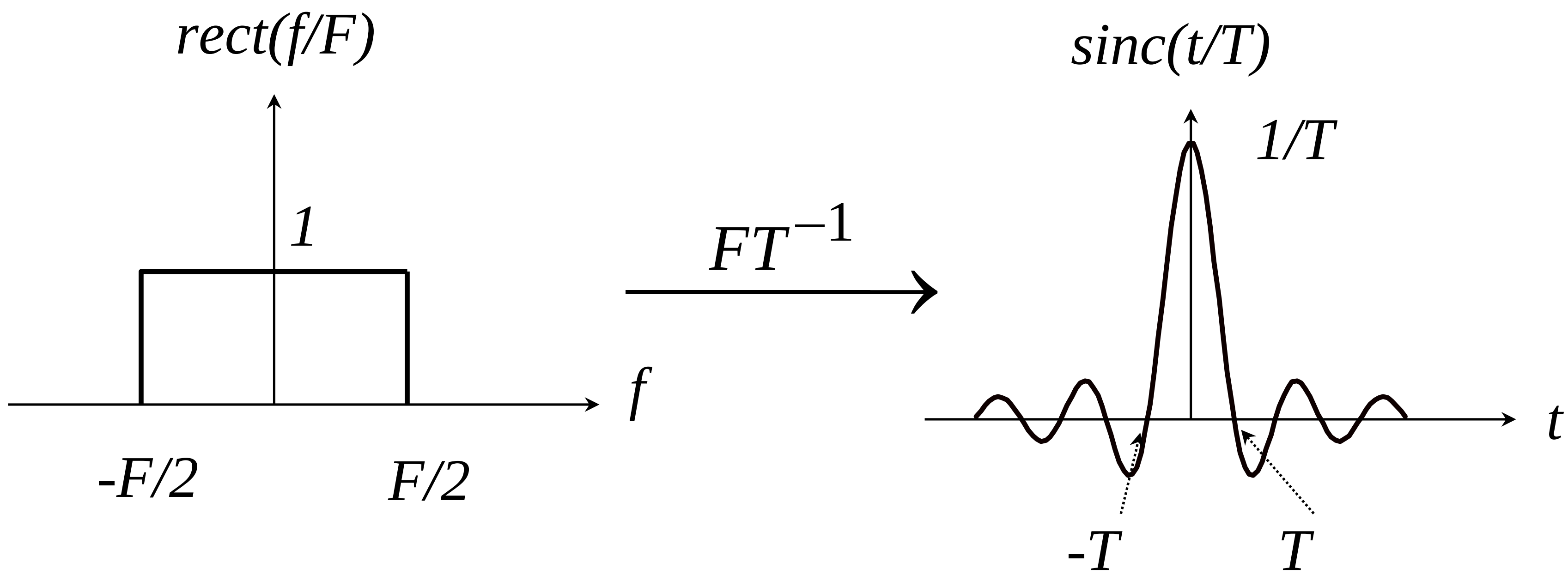
Principes



Aparté 1
(Théorème de convolution)

$$\int_{-\infty}^{+\infty} x(\tau)y(t-\tau)d\tau \;\underset{TF^{-1}}{\overset{TF}{\rightleftarrows}}\; X(f).Y(f)$$

Aparté 2



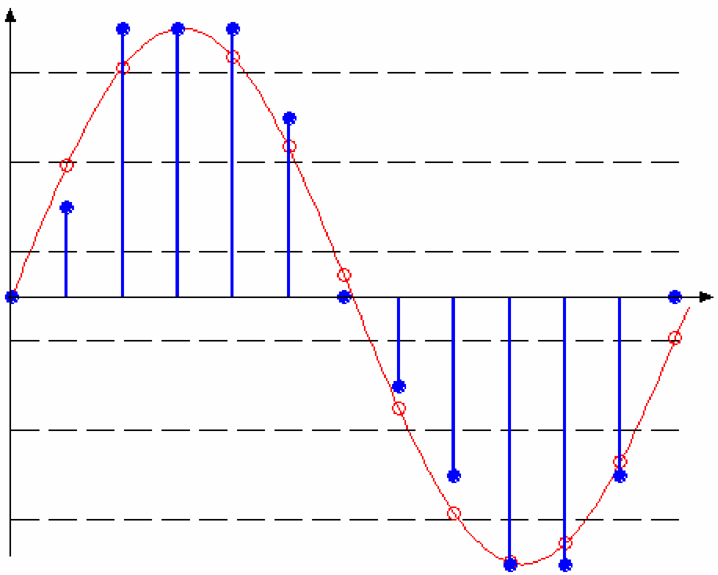
$$rect\left(f/F\right)=\begin{cases}1 & \text{if} \quad \left|f\right|<F/2\\ 0 & \text{else.}\end{cases}$$

$$\mathrm{sinc}(t/T)=\frac{\sin(\pi t/T)}{\pi t}$$

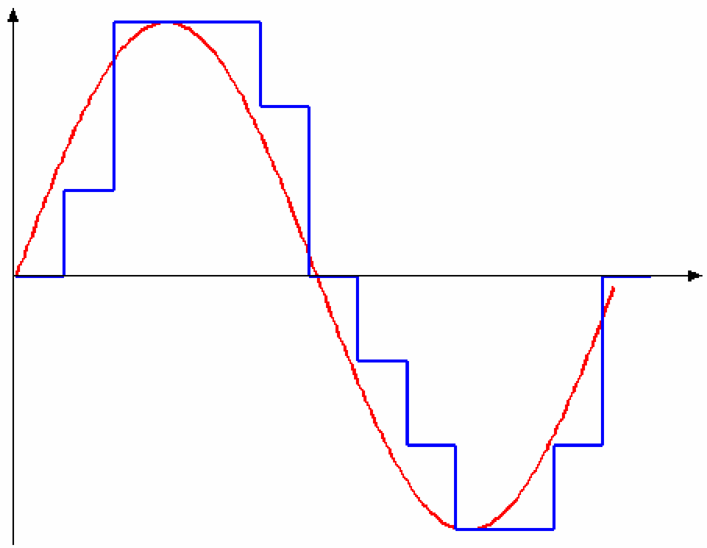
Du numérique à l’analogique

000
001
011
011
011
010
000
101
110
…

Décodage

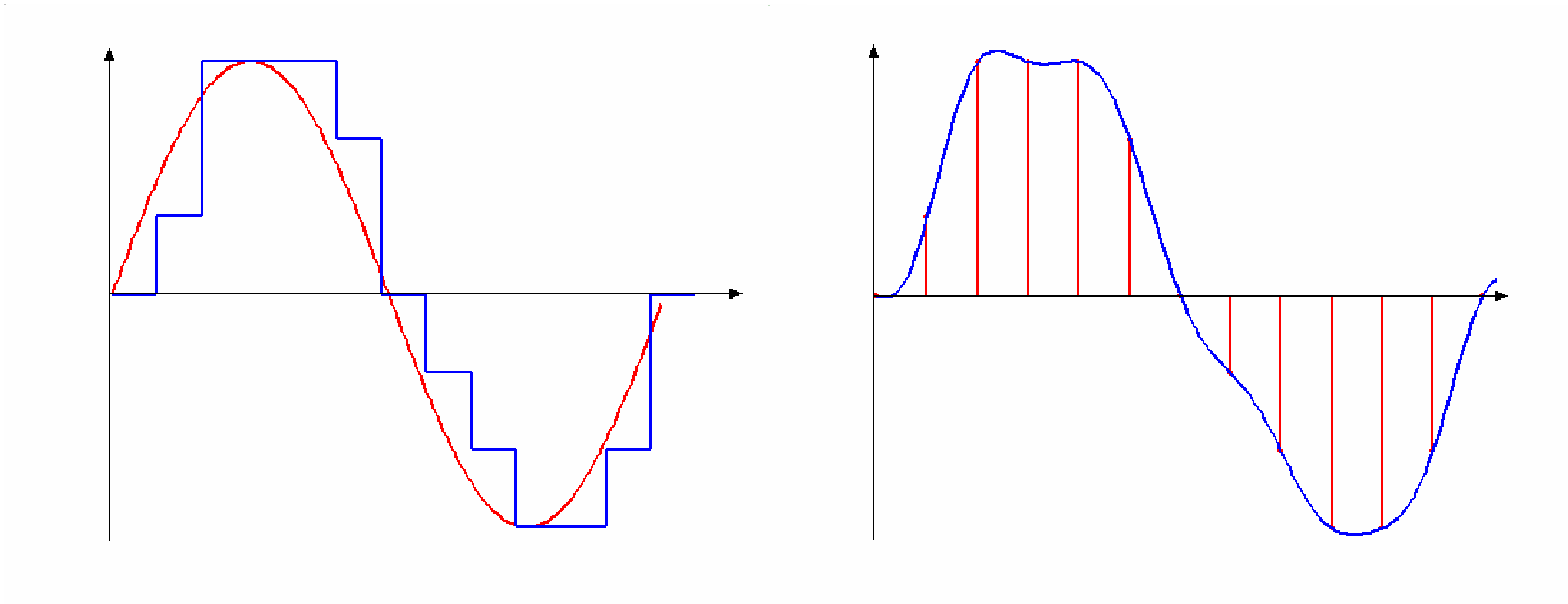


Blocage

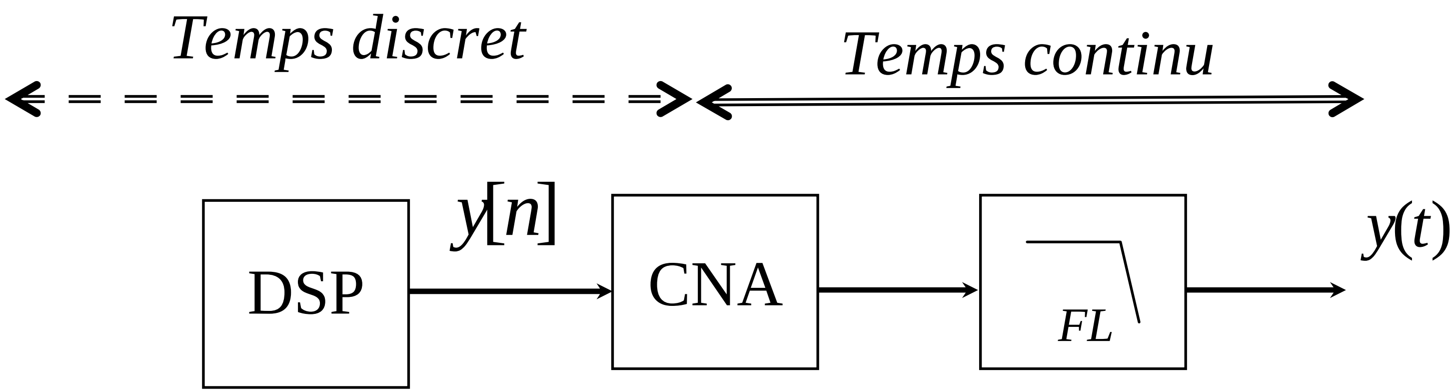


…(suite)

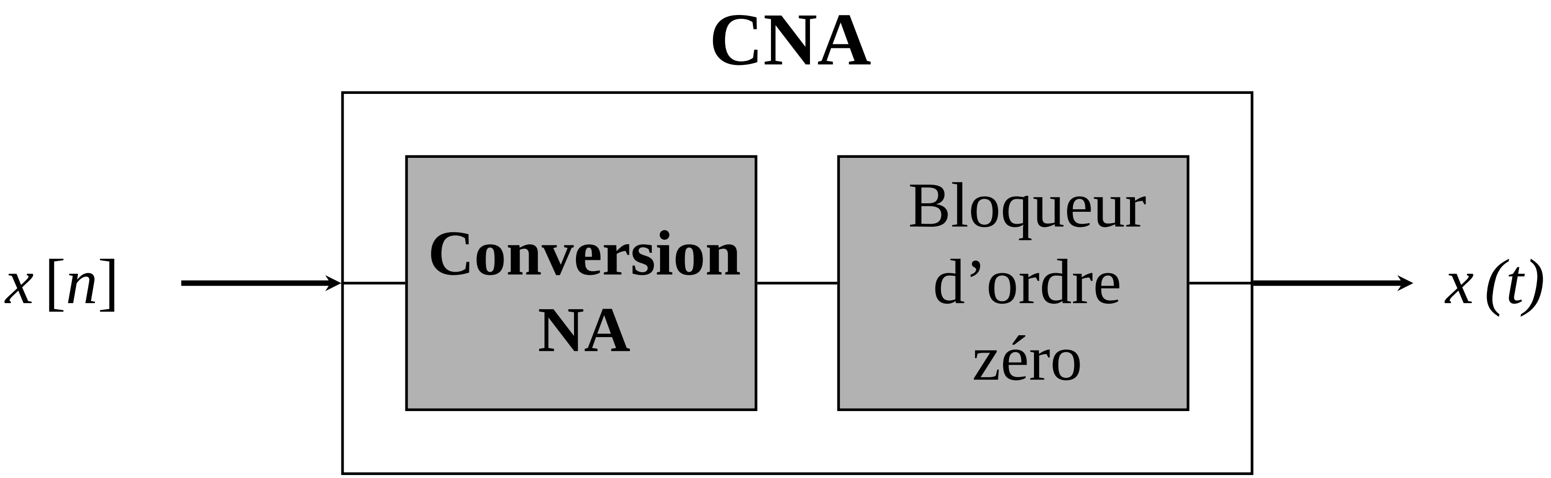
Lissage

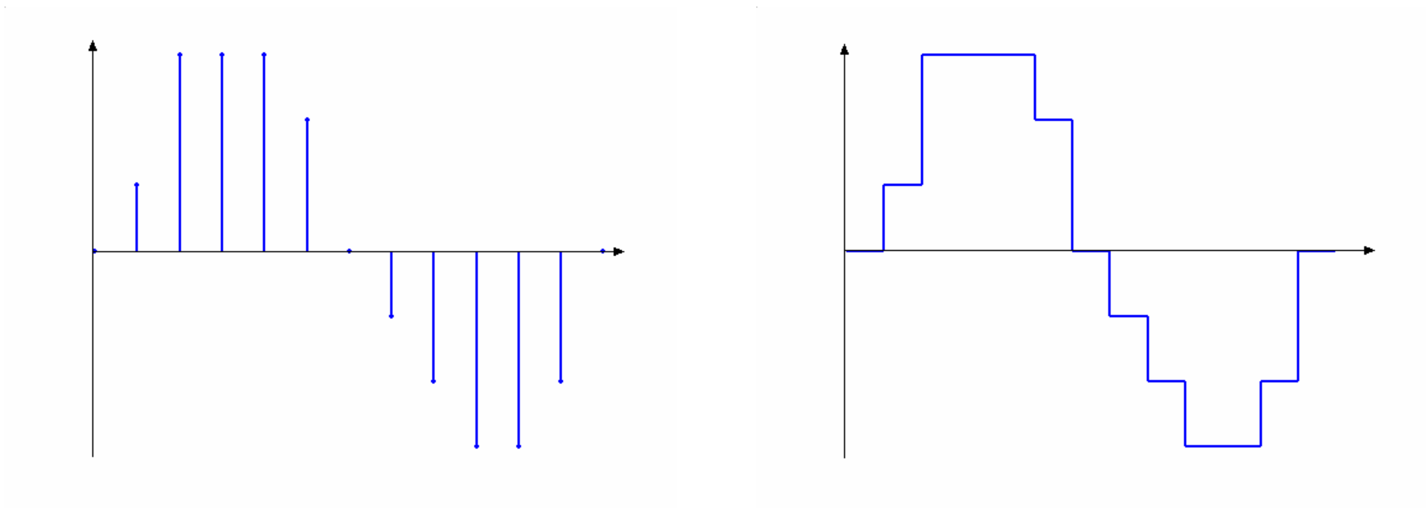


Chaîne de traitement

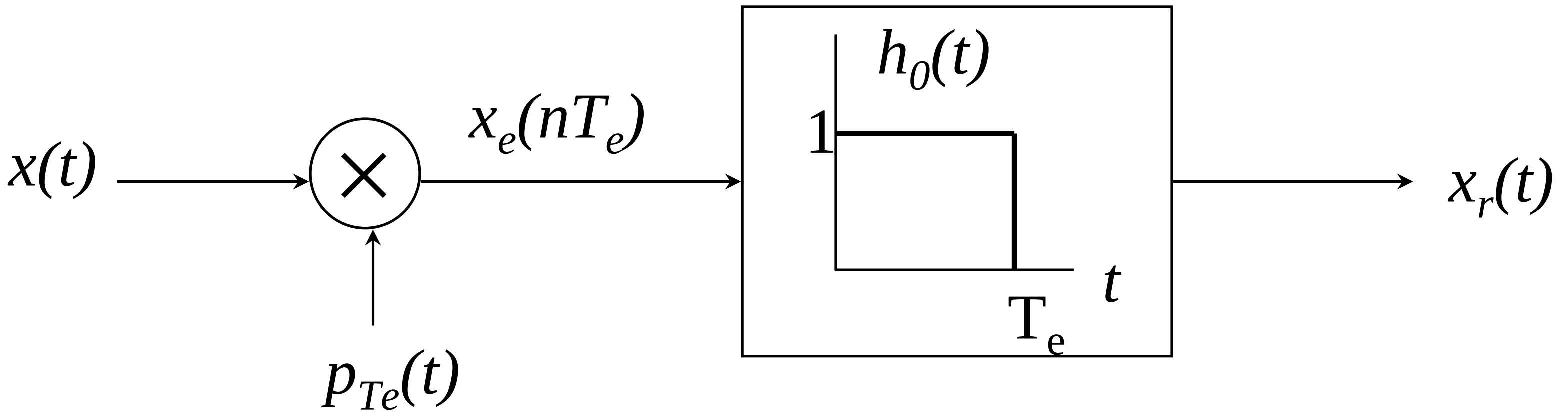


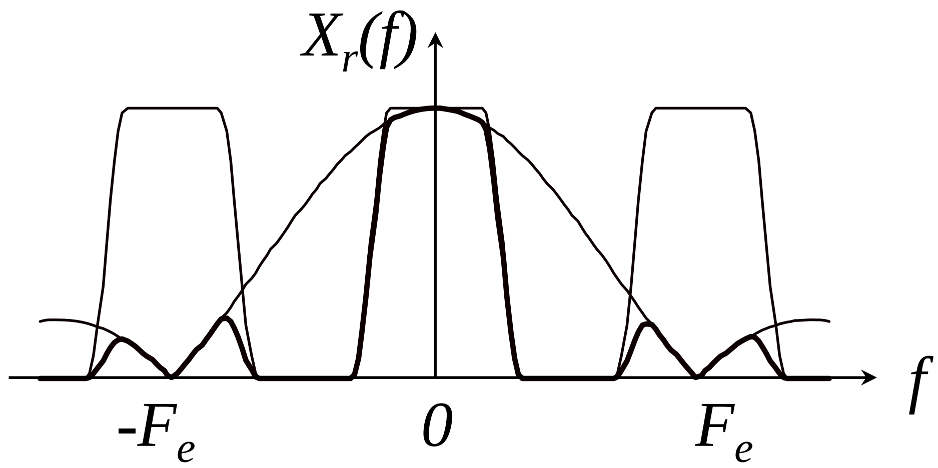
Convertisseur Numérique analogique





…(suite)

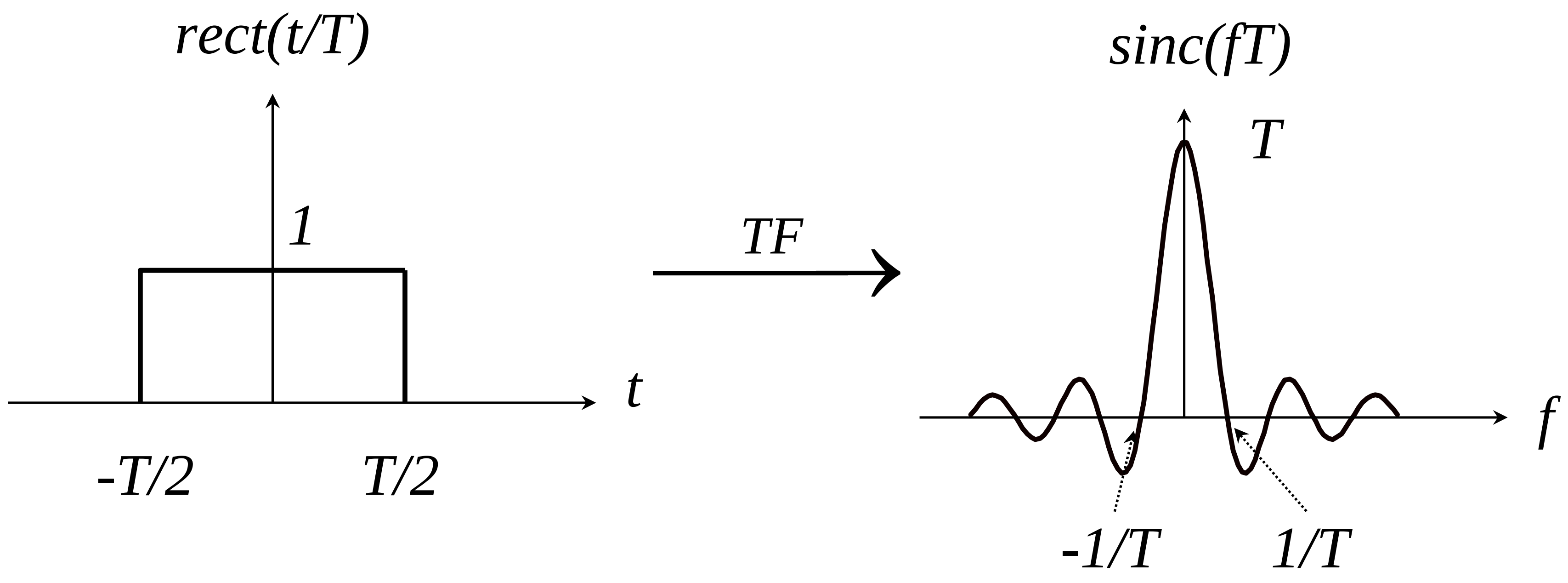




Aparté 1
(Théorème de convolution)

$$\int_{-\infty}^{+\infty} x(\tau)y(t-\tau)d\tau \; \underset{TF^{-1}}{\overset{TF}{\rightleftarrows}} \; X(f).Y(f)$$

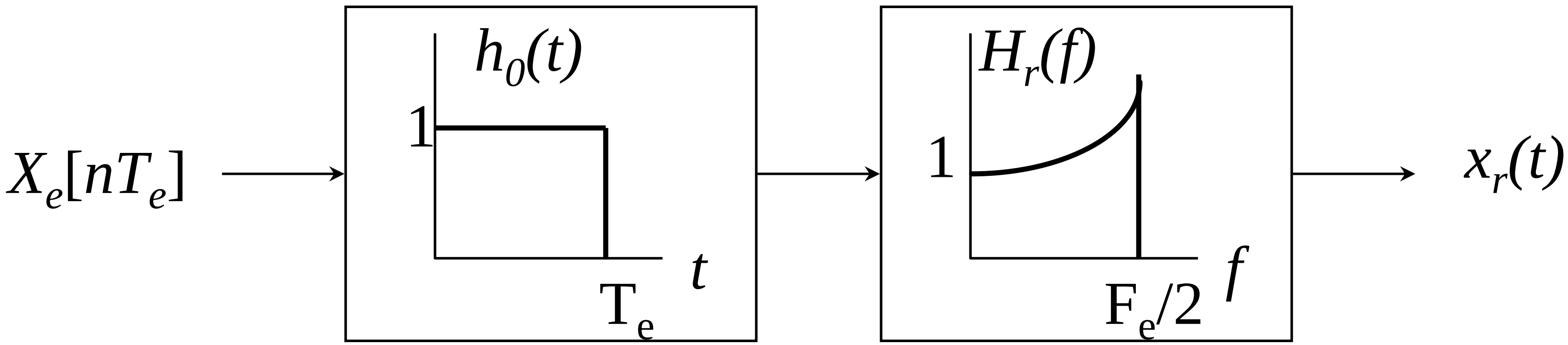
Aparté 2
(Fonctions rectangle et sinus cardinal)



$$rect\left(t/T\right)=\begin{cases}1 & \text{si} \quad |t|<T/2,\\ 0 & \text{sinon.}\end{cases}$$

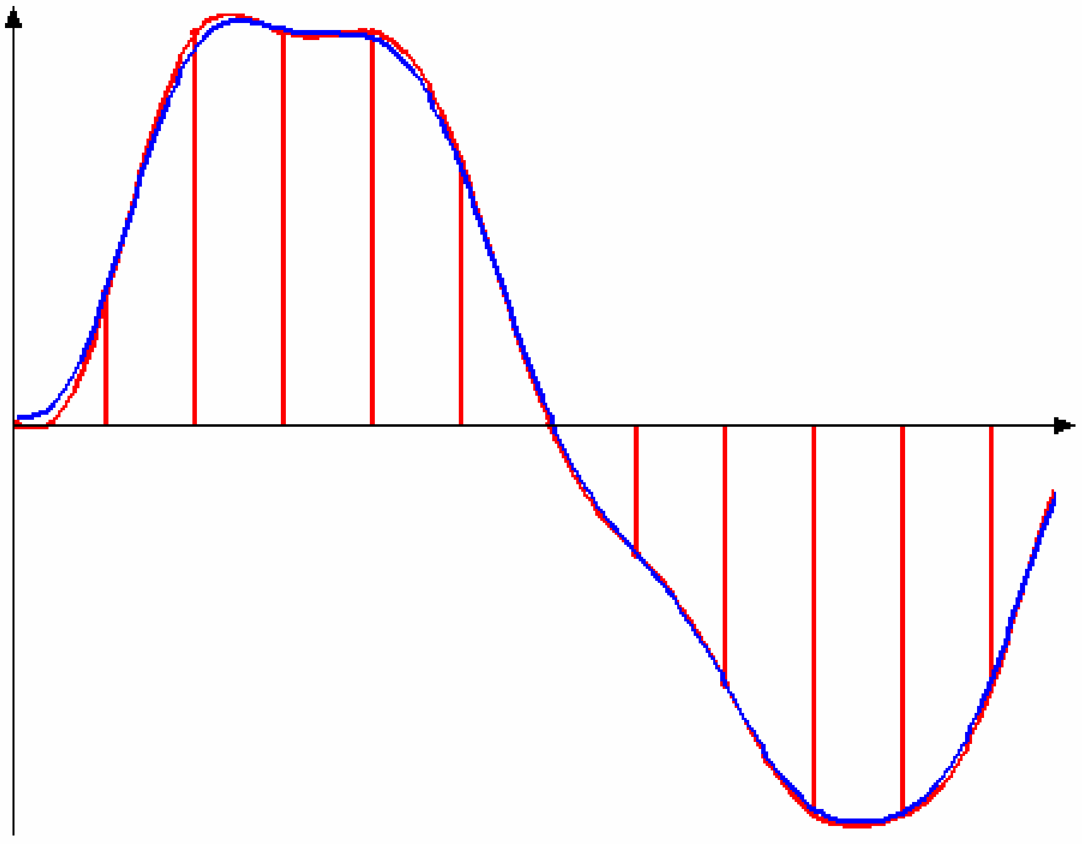
$$\text{sinc}(fT)=\frac{\sin(\pi fT)}{\pi f}$$

Filtre de lissage



$$H_r(f) = \begin{cases} \dfrac{1}{\mathrm{sinc}(\mathrm{T_e}\, f)} & \text{pour } |f| \leq F_e/2, \\ 0 & \text{sinon.} \end{cases}$$

…(suite)



Abaques

- La correction en 1/sinc peut être approximée par la surtension d'une fonction de transfert d'ordre supérieur à 1